Tab 3: Numerical Fluency Introduction

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Tab 3: Numerical Fluency Introduction
Master Materials List

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Scratch paper

The following materials are not within this tab of the notebook, but they can be accessed by clicking on the links below.

Resources for TEKS activities: K-5 TEKS Refinements Banner, Grade Level Posters, and K-2 and 3-5 Enlarged TEKS Introductory Statements

Numerical Fluency Definition Poster

Slides 1-17, Numerical Fluency PowerPoint
Activity: Defining Numerical Fluency

(3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.

111.15, 16, 17. Mathematics, Grades 3-5. Introduction.
(3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.

Overview: The entire Trainer of Trainers numerical fluency presentation was developed to be presented in a 9-hour session. The purpose of this professional development is to make participants aware of the refinements that have been made to the grades K-5 Mathematics Texas Essential Knowledge and Skills (TEKS). After each activity refer back to the refined TEKS that were covered. This allows the participants to keep focused on the purpose of the training. The thought is to start with a problem that could possibly be given to fifth graders and then look back at what needs to be taught at each grade level to lay the foundation for successful numerical fluency. Participants will focus on the patterns and relationships that exist within sets of number and among operations. Many of the activities will attempt to develop new vocabulary. It is hoped that with some activities, the activities build on prior knowledge and advance the participants understandings of those concepts.

As the first section of the professional development, this activity defines numerical fluency and will have participants reflect on their own developmental level of numerical fluency. Many of the participants will have attended the state reading academies and will be familiar with the definition of reading fluency. It was the attempt of the writers to connect that prior knowledge with a working definition of numerical fluency.
This activity was developed to be a lecture type presentation with some participation from the participants. It is very important for the remainder of the professional development that participants have an understanding of numerical fluency.

Realizing that trainers will often need to provide professional development for teachers from different grade level configurations, these materials have been developed to be easily adapted for sessions involving only K-2 teachers, only 3-5 teachers, or combined groups of K-5 teachers. Trainers need to consider teachers’ experiences as well as their understanding of the foundational nature of these concepts when making decisions on how much emphasis is needed for teachers to meet their students’ needs. Suggestions related to these possible adaptations will be included in trainer notes.

**Materials:** Slides 1-17, Numerical Fluency PowerPoint

Scratch paper

Resources for TEKS activities: K-5 TEKS Refinements Banner, Grade Level Posters, and K-2 and 3-5 Enlarged TEKS Introductory Statements

Index Cards

**Grouping:** Groups of 4-6 based on grade levels.

**Time:** One hour

**Lesson:** Distribute copies of the PowerPoint slides 1-17 to participants to help them focus and take notes on the important ideas from the PowerPoint presentation. Use the following notes to elaborate on the content of each slide, 1-17, of the Numerical Fluency PowerPoint.

<table>
<thead>
<tr>
<th>Procedures</th>
<th>Notes</th>
</tr>
</thead>
</table>
| **Slide 1**<br>Building Foundations for Mathematics<br>Defining Numerical Fluency | Post K-5 TEKS Refinements Banner on the wall with Grade Level Posters immediately below. (See materials list for links to these resources.)

This wall area needs to be large because participants will post refined TEKS throughout the professional development. You may want to have a small version of the TEKS that have not been refined to place on the wall as well. To emphasize the refinements, only the refined TEKS are enlarged. |
Goals & Purposes

- Increase teacher knowledge regarding the refinements of the TEKS relating to numerical fluency.
- Develop a working definition of numerical fluency.
- Increase teacher knowledge of composing and decomposing numbers.
- Increase teacher knowledge of developmental stages of numerical fluency.
- Increase teacher knowledge of strategies to develop numerical fluency.
- Develop an understanding of the use of metacognition in problem solving.

Share with participants the goals and purposes of this training.

Research (Fennema & Franke, 1992) shows a high correlation between teacher knowledge and student knowledge and thus success in mathematics.

As you go through each topic, keep in mind that this approach facilitates learning for ELL and other students with special needs. Building a strong conceptual foundation of numerical fluency is important for all students' success and especially critical for ELL and other students with special needs.

Click once to display the word “Math” on the PowerPoint slide and ask participants: How would you define mathematics?

Give participants time to think about their definition and then have them share their definitions with the person next to them.

Then ask:
(1) How many defined math numerically?
(2) How many defined it geometrically?
(3) How many defined it as looking at patterns and relationships?

Indicate you want to start with “Just patterns waiting to be found” as the definition of mathematics. (Display entire slide by clicking twice more as you say the previous statement.) This Haiku was written by a 3rd grader. This definition of mathematics is necessary for us to understand that mathematics is “alive.” It allows one to view mathematical problems from various perspectives. It also allows us to look at numbers and operations from a different perspective.
### Procedures

<table>
<thead>
<tr>
<th>Slide 4</th>
<th>Texas Essential Knowledge and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.</td>
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</tr>
</tbody>
</table>

### Notes

Now, with our understanding of mathematics as looking at patterns and relationships, have participants go back and look at the refinements of the TEKS for mathematics. Post the K-2 enlarged introductory statement below the K, 1, & 2 posters. (See materials list for link to enlarged version.) In the K-5 introductory paragraphs statement 3, the educators of Texas believe that numerical fluency must be developed.

This part of the training focuses on numerical fluency of grades K-5 students and provides only a glimpse of what they need to become numerically fluent. Topics addressed are based on research and input from educators across the state as being necessary for foundations of numerical fluency.

Ask participants what this introductory statement means if mathematics is defined as patterns and relationships.

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### Procedures

<table>
<thead>
<tr>
<th>Slide 5</th>
<th>Texas Essential Knowledge and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.</td>
<td></td>
</tr>
</tbody>
</table>

### Notes

On the wall below the grades 3-5 posters, place the enlarged version of the Grades 3-5 TEKS Introductory Statement 3. (See materials list for link to enlarged version.)

Now that the K-2 Introductory Statement 3 has been dissected, ask participants what does the Grades 3-5 Introductory Statement 3 mean if mathematics is defined in terms of patterns and relationships?

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### Procedures

<table>
<thead>
<tr>
<th>Slide 6</th>
<th>How Do You Use Numerical Fluency?</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Solve the following problem mentally.</td>
<td></td>
</tr>
<tr>
<td>• Ms. Hill wants to carpet her rectangular living room, which measures 14 feet by 11 feet. If the carpet she wants to purchase costs $1.50 per square foot, including tax, how much will it cost to carpet her living room?</td>
<td></td>
</tr>
<tr>
<td>• Write down your thought processes of how you solved the problem.</td>
<td></td>
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<tr>
<td>• Turn to someone next to you and share your problem solving strategies.</td>
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</tr>
</tbody>
</table>

### Notes

Note: This problem is presented to focus on participants’ numerical fluency and is in no way indicative of student expectations for K-5 students. This question was taken from the 2004 Released Ninth Grade TAKS Test, question 49.


Have participants solve this problem mentally. Only give 30 seconds for the problem to be read and solved. Time is limited to help participants reflect on their own numerical fluency.
<table>
<thead>
<tr>
<th>Procedures</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>This is much like the warm up problems that are given at the beginning of class time. After participants solve the problem, have them write down their thought process in solving the problem and share their strategies with others. Many possible strategies other than the traditional algorithms can be used to solve this problem. Some of these strategies are:</td>
<td></td>
</tr>
<tr>
<td>(1) $14 \times 11 = 154; \quad 154 \times 1 = 154; \quad 154 \times \frac{1}{2} = 77; \quad 154 + 77 = 231$</td>
<td></td>
</tr>
<tr>
<td>(2) $14 \times 10 = 140; \quad 140 \times 3 = 420; \quad 420 / 2 = 210; \quad (14 \times 3) / 2 = 21; \quad 210 + 21 = 231$</td>
<td></td>
</tr>
<tr>
<td>(3) $14 \times 11 = 154; \quad 154 / 2 = 77; \quad 77 \times 3 = 231$</td>
<td></td>
</tr>
<tr>
<td>(4) $[(14 \times 3) / 2] \times 11 = 231$</td>
<td></td>
</tr>
</tbody>
</table>

Looking back at Introductory statement #3 for K-5, the statement discusses composing and decomposing numbers. The example strategies used to solve this problem involved composing and decomposing numbers. Based on these examples, let us define composing and decomposing numbers.

**Slide 7**

**Composing and Decomposing**

- Building and taking apart numbers
- Looking for patterns/relationships between numbers
- Unitizing numbers
- Using numbers as reference points

Based on our discussion, we then can say that **composing and decomposing** is the ability to build and take apart numbers based on relationships and patterns of numbers.

Composing and Decomposing numbers relies on the ability to unitize numbers. According to Fosnot and Dolk (2001b), unitizing is the ability to look at quantities as units like we do in base ten. But base ten is not the only example of unitizing. Anytime you group things and look at a group as one set, then you are unitizing. Therefore when you say 1 set of 2 = 2 and two sets of 2 = 4, you are unitizing. This approach leads to the understanding of multiplication. It also helps in the development of adding on or counting down. Unitizing is a huge part of the
Using numbers for reference points (10, ½, 100, 5, 20, 25) is important in being able to decompose and compose numbers quickly by creating compatible numbers that are easily manipulated. This allows students to become fluent with numbers (NCTM, 2000).

Our ultimate goal is for our children to have numerical fluency. But what is numerical fluency?

We begin trying to define numerical fluency by looking at the definition of number sense.

Howden (1989) defined number sense as “good intuition about numbers and their relationships." That means being able “to decompose numbers naturally, use particular numbers like 100 or ½ as referents, use the relationships among arithmetic operations to solve problems, understand the base-ten number system, estimate, make sense of numbers, and recognize the relative and absolute magnitude of numbers. Number sense develops gradually as a result of exploring numbers, visualizing them in a variety of contexts and relating them in ways that are not limited by traditional algorithms.” (NCTM, 2000, p. 32). Students are able to utilize strategies (doubles, doubles plus one, anchoring to ten, and numerous other strategies).

But what about using these skills to solve problems? If you look back at the introductory paragraph though, you will see that it talks about solving problems. What does this mean?

<table>
<thead>
<tr>
<th>Procedures</th>
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</thead>
<tbody>
<tr>
<td>foundation of number sense for K-5 students.</td>
<td></td>
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</tbody>
</table>

**Slide 8**

**Definition of Number Sense**

- Number sense is a “…good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts and relating them in ways that are not limited by traditional algorithms (p. 11).”

### Slide 9

**Problem Solving**

- The potential to model (use manipulatives, draw pictures, create tables, charts, or graphs) the situation must be a natural progression of the problem.
- The problem needs to be well defined so that children can analyze the problem and comprehend what they are to accomplish.
- The problem encourages children to delve deeper into the problem by asking questions and identifying patterns.


### Slide 10

**Reading Fluency**

Fluency is important because it provides a bridge between word recognition and comprehension. Because fluent readers do not have to concentrate on decoding the words, they can focus their attention on what the text means. They can make connections among the ideas in the text and between the text and their background knowledge. In other words, fluent readers recognize words and comprehend at the same time. Less fluent readers, however, must focus their attention on figuring out the words, leaving them little attention for understanding the text.


Ask how many participants have attended the state reading academies. The National Institute for Reading defines reading fluency as a bridge between word recognition and comprehension. So how can we make a transition from this definition to numerical fluency?

Highlighted words are important. Read: “Fluent readers recognize words and comprehend at the same time. Less fluent readers must focus their attention on figuring out the words, leaving them little attention for understanding the text.”

### Slide 11

**Numerical Fluency**

Fluency is important because it provides a bridge between number recognition and problem solving comprehension. Because people who are numerically fluent do not have to concentrate on operation facts, they can focus their attention on what the problem means. They can make connections among the ideas in the problem and their background knowledge. In other words, people who are numerically fluent recognize how to compose and decompose numbers based on patterns and comprehend how to use those numerical patterns to solve problems. People who are less fluent, however, must focus their attention on the operations, leaving them little attention for understanding the problem.


In an effort to make a connection between reading and numerical fluency, this definition was developed for numerical fluency: Go over definition on slide.

Numerical fluency provides a bridge between number recognition and problem solving comprehension.

People who are numerically fluent, recognize how to compose and decompose numbers based on patterns and comprehend how to use those numerical patterns to solve problems. It is more than just solving abstract problems and spitting out answers. Solving problems must be within meaningful contexts.
### Procedures

**Slide 12**

**Development of Numerical Fluency**

- First the student MUST build an understanding of composing and decomposing number through meaningful problems.
- Then through much practice, children build automatically, which is the fast, effortless composing and decomposing of numbers.

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**Notes**

A child who is numerically fluent can solve problems in multiple ways and communicate those solutions to others.

A child who possesses only automaticity can solve algorithms mindlessly, without much regard for the relationship between the numbers involved. Thus, automaticity is a necessary part of fluency, but is not sufficient by itself.

Now, what does the state say about developing automaticity? At this time in your table groups look through the grade level TEKS and find where it requires speed when working with numbers. Jot down all the times students are required to master concepts based on speed. Give a few minutes for participants to identify TEKS. *There will be none.*

Thus, the state requires that the students be efficient – that is proficient and be able to quickly compose and decompose numbers. In business if a worker is efficient, they do not waste time in producing a product for a company. Thus, numerical fluency is more involved/complicated than automaticity and number sense, but involves both conceptual ideas. It is a complex web of concepts and strategies used to solve meaningful problems within real-world contexts with efficiency and ease.

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**Slide 13**

**Numerical Fluency**

Numerical Fluency is the ability to compose and decompose numbers flexibly, efficiently, and accurately within the context of meaningful situations.

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**Notes**

Therefore we can define numerical fluency as the ability to compose and decompose numbers flexibly, efficiently, and accurately within the context of meaningful situations. [Trainers can print this definition as a poster, using this link, and refer to it throughout the presentation.]

How are number sense and numerical fluency related? Numerical Fluency is your final product. Number sense is part, but not all, of...
<table>
<thead>
<tr>
<th>Procedures</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>the building blocks toward numerical fluency. Numerical fluency differs from number sense in that we are placing composing and decomposing within the context of meaningful situations. We also feel those who are numerically fluent can work with numbers efficiently and accurately. In other words, the students possess automaticity with their addition, subtraction, multiplication, and division facts. For instance, students should be able to quickly tell you many possible combinations of composing and decomposing 7. In our carpet problem, we expected you to work the problem mentally using strategies that you have learned within a short period of time. We were trying to engage you in a problem that would allow you to demonstrate your numerical fluency. Kamii and Dominick (1998) found that children who see problems horizontally will tend to use mental math and other strategies more than those who see the problems vertically. Those who see the problems vertically tend to use the traditional algorithm.</td>
<td></td>
</tr>
</tbody>
</table>

Reflect on your own numerical fluency. On an index card, write yourself a short note regarding your numerical fluency. We will come back to this card several times over the course of the training.
<table>
<thead>
<tr>
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<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Slide 15</strong></td>
<td>Now, let us look at a student’s numerical fluency. Abby, a fourth grader, was given two problems: 8+8= and 8+9=. She was asked which of these two problems had the largest sum. Abby quickly responded that the answers were 16 and 17. When she was asked which is the larger sum, she responded with 17. She was then asked: “How do you know that sum is larger?” Abby responded by saying: “Because I know my facts.” She was asked in several ways to explain how she knew that 8 + 9 = 17 was larger than 8 + 8 = 16. Each time she responded with “Because I know my facts.” What do you think about Abby’s numerical fluency? Think back on our definition. Did she possess the ability to compose and decompose numbers? Did she understand the patterns or relationships between numbers in the problem? Does she possess the ability to unitize numbers or is there enough information to know? As you go off on a 10 minute break, discuss with others Abby’s numerical fluency.</td>
</tr>
</tbody>
</table>

| **Slide 16** | We have established a pattern for looking at students’ numerical fluency. We will spend the rest of the day looking at the development of understanding numbers – patterns and relationships, how to compose & decompose numbers, and how these fit into the operations. As we take a 10-minute break, look for patterns that occur in our daily lives. When you come back, please sit in groups based on grade level. Note: The timer on this slide is set for 10 minutes. |

**Abby**

8+8=
8+9=

How would you describe Abby’s numerical fluency?
Inform the participants that they have 12 seconds before you begin again. This clock clicks off 12 seconds. Participants should be back in seats ready to begin when the clock stops ticking.

Resources:

Texas Education Agency (2006). Texas essential knowledge and skills online at http://www.tea.state.tx.us/teks