Foundations for Functions

Activity: The Hand Squeeze

TEKS: (A.2) **Foundations for functions.** The student uses the properties and attributes of functions.

The student is expected to:

(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

(D) collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.

Overview: In this activity, students will collect data and organize it in order to analyze the relationship between the number of participants and the time it takes to "pass" a hand-squeeze around a circle. They will also explore the differences between discrete and continuous variables to refine their understanding of these topics.

Pre-requisites: Students have:

(1) had experience with creating scatterplots
(2) learned some of the characteristics of linear functions
(3) been exposed to the concept of correlation.

Vocabulary: Linear, correlation, data, discrete data, scatter plot, independent variable, dependent variable, domain, interpolation, extrapolation, and slope

Materials: *Data Recording Sheet* transparency (or poster)
Transparency (or poster) markers
Stopwatch (or watch with time-keeping ability)
Quarter-inch or centimeter grid paper (one sheet for each student)
Analyze the Data handout

Grouping: Collection of data and discussion at the end of activity should be a whole class activity, but the analysis of data should be conducted in groups of two or three students.

Time: 45 to 60 minutes

Lesson:

Procedures	Notes
1. Select a time-keeper and a recorder for the class. Give the time-keeper the stopwatch (and make sure he/she knows how to	Teacher needs to decide how accurately the time should be measured (i.e. to the nearest

	operate it), and give the recorder the recording sheet or poster and markers. Also select the first two participants.	tenth, hundredth, or thousandth of a second, depending on the stopwatch's capabilities).
2.	**Describe the activity to the students:** We will time how long it takes to "pass" a hand squeeze all the way around a circle. We will begin with two students in our circle, and then add two students to the circle for each additional trial. When the timer says "Go," the first person will squeeze the hand of the person on his/her right; each successive participant will squeeze with the right hand as soon as he/she "receives" the squeeze on his/her left hand. When the first person feels the squeeze on his/her left hand, he/she will say, "Now!" to stop the time.	Allow two or three students to practice and model the activity to the class and for the timer to practice using the stopwatch.
3.	**Perform the trials:** Time the hand squeeze for the first two participants and have the recorder record the time on the recording sheet. Then add two more participants and repeat the squeeze. Keep adding two participants and recording the time until all students (except the time-keeper and the recorder) are in the circle.	Make sure that the time-keeper is consistent in the way he/she keeps time. Direct students to pass the hand squeeze as quickly as they feel it, and to pass it the same way every time so that the data is not too distorted. If someone messes up, it is okay to disregard that and repeat the trial.
4.	Distribute grid paper and direct students to graph the data from the class recording sheet. Ask: "Which variable should we use for the independent variable? Why?" "Which variable should we use for the dependent variable? Why?" *The amount of time is dependent upon the*	Allow students to work in pairs, if needed, but be sure that students understand that each one should create his/her own graph.

	number of students. The number of students is the independent variable, x, and the number of seconds is the dependent variable, y.	
5.	Distribute the handout, *Analyze the Data*. Students should work in groups of two or three to answer the questions.	Circulate during this time to clarify terms such as discrete data, correlation, increasing/decreasing, etc.
6.	Have the groups share their answers, and discuss them as a class.	Be sure to bring out discussions of discrete vs. continuous data, correlation, and the concept of a constant rate of change.

Homework: Have students collect some kind of bivariate (data that relates two variables such as time and distance or in this case number of students and time to complete the squeeze) data at home, graph it, and answer the same questions as on their handouts. The data collection may require more than a single night; you may want to make this a project.

Assessment: Assessment is primarily during the analysis part and from homework.

Extensions: Question 5 on the *Analysis of Data* handout asks students to predict how many seconds it would take to pass the squeeze around a circle of 100 people. When discussing their prediction for the time for the 100^{th} squeeze, the teacher can introduce the terms extrapolation and interpolation. The 100^{th} squeeze is a value of x that falls outside the range of the data which makes it an extrapolation. Have students choose a value of the x that would be an interpolation. Ask students whether they think extrapolation or interpolation would be more useful to do if they created a model for data.

Using the data collected in the class activity and students' knowledge of linear equations, have students come up with an equation of a line they feel fits the data. The teacher can then demonstrate using the "squeeze data" how to create scatterplots on a graphing calculator and how to find the least-square regression line.

Look up the distances to different places, such as Mexico City, Panama City, Toronto, Buenos Aires, etc. Decide how many children it would take to stand holding hands from here to there. Then predict how long, based on their models, it would take to pass a hand squeeze all the way from here to there.

There are a number of other movements that could be used in place of the hand squeeze; examples include a rifle drill team, flag corps, cheerleading competition, the wave around a stadium, etc.; teachers could use video tapes of different sized squads of any of these movements and time how long it takes to "pass" the movement from one end of the squad to the other. Students could also be asked to time the movement of a single squad, and then interpolate or extrapolate the data to predict how long it would take 2, 4, 6, etc. members to "pass" the same movement. They could also substitute "the wave" for the hand squeeze, and then use the data to predict how long it would take the fans in a stadium to complete one circuit of "the wave" around the stadium. The stadium could be selected from a professional or college sport, or from the local school district or the home field.

Resources: This activity was adapted from The Hand Squeeze, by Cynthia Lanius, as posted on her website, http://math.rice.edu/~lanius/Algebra/hndsq.html.

Data Recording Sheet Transparency

Number of students	Number of seconds
2	
4	
6	
8	
10	
12	
14	
16	
18	
20	
22	
24	
26	
28	
30	
32	

Analyze the Data

Answer each of the following questions for the data you have collected as a class:

1. Should you connect the points on your graph to make a solid line? Why or why not? What is the domain of this situation?

2. Describe the relationship between the number of students and the number of seconds it takes to pass a hand squeeze.

3. The data follow the pattern or shape of a line. Draw in a line that seems to fit most of the data. What about the data makes it linear?

4. What kind of correlation is there (if any)? (A positive correlation exists if the data points generally trend upwards from left to right. Negative correlation exists if the data points generally trend downwards from left to right; and there is no correlation if either the data are not generally linear or there is no real trend either upwards or downwards from left to right.)

5. Make a prediction: Based upon the data collected, how many seconds would it take to pass the hand squeeze around a circle of 100 people? How many minutes would it take? Explain your reasoning.

Analyze the Data

Sample Solutions

Answer each of the following questions for the data you have collected as a class:

1. Should you connect the points on your graph to make a solid line? Why or why not? What is the domain of this situation?
   It is not appropriate to connect the points here; the data is discrete. i.e., the number of students cannot be 2.5 or 5.9; thus any values that are not whole numbers are inappropriate to use for the independent variable.
   Based upon not having fractional values for students, the domain for this situation should be integers greater than zero.

2. Describe the relationship between the number of students and the number of seconds it takes to pass a hand squeeze.
   The higher the number of students, the longer it takes to pass a hand squeeze around the circle.

3. The data follows the pattern or shape of a line. Draw in a line that seems to fit the data. What about the data makes it linear?
   The data are generally linear, as they tend to fall along an upward sloping line. The time on average increases about the same amount for each additional student. The amount the time changes for each student would represent the slope of the line.

4. What kind of correlation is there (if any)? (A positive correlation exists if the data points generally trend upwards from left to right. Negative correlation exists if the data points generally trend downwards from left to right; and there is no correlation if either the data are not generally linear or there is no real trend either upwards or downwards from left to right.)
   There is a positive correlation: as the number of students increases, so does the time it takes to pass the hand squeeze. Teachers, you

may want to also address here the idea of increasing and decreasing functions and tie that to correlation in generally linear data. Also you want to emphasize the connection that a positive correlation means that the line will have a positive slope or rate of change.

5. Make a prediction: Based upon the data collected, how long would it take to pass the hand squeeze around a circle of 100 people? How many minutes would it take? Explain your reasoning.
Answers will vary. Remind students that there are 60 seconds in a minute, and if necessary, review changing units. Explanations will vary. If the data fits on a line that does not appear to contain the origin, the problem should not be solved using proportions. To use a proportion, the situation has to be directly proportional (Line goes through the origin).