# Work

We are often are asked to find the speed of an object in a physics problem.

We could use Newton's Laws to find acceleration and then try and find velocity. Once we found the velocity, we could find the vector's magnitude to find the speed.

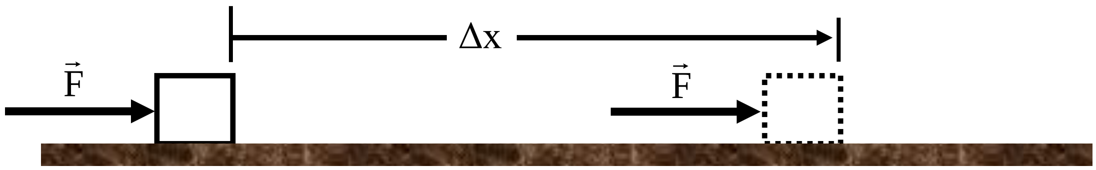
While a valid approach, there are many math difficulties with this approach. One difficulty is that forces are vectors which usually means considerable trigonometry to solve the acceleration especially if the forces change direction during the motion of the object. Also the process of getting from acceleration to velocity is often difficult mathematically except for easier problems where acceleration is constant and we have the kinematic equation or for problems where the area under the acceleration-time graph is a shape that we recognize. Another difficulty that arises in some problems is the case where we don't know one or more of the forces in which case we can't find the acceleration.

While a force causes a particle to accelerate, this doesn't mean that it always causes an object to change speed. We saw an example of this in uniform circular motion where a force caused acceleration but the object's speed was constant. If a force doesn't cause a change in the speed of the particle then perhaps we can find a method by which we can this force.

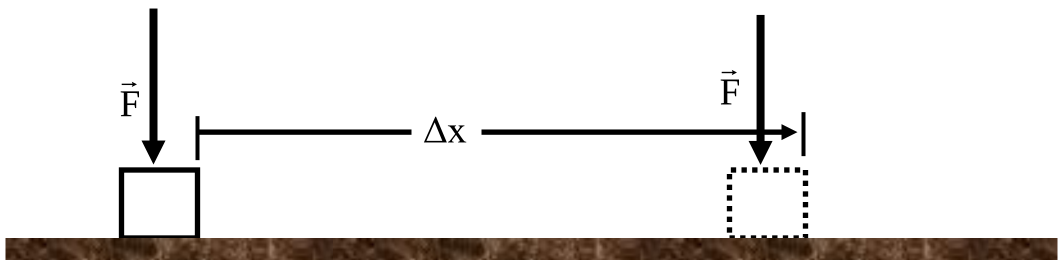
Thus, we look for a new concept that more directly ties the application of a force to the change in the speed of an object. This new concept is called work.

**I.** **Work**

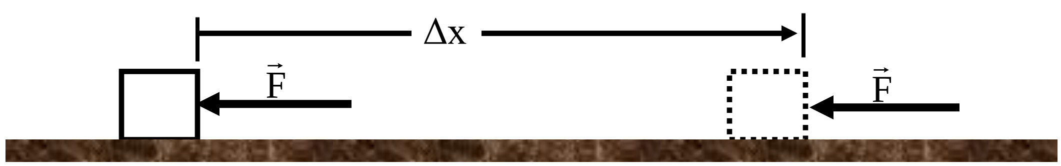
In the diagrams below a force F is applied to a block as the block is moved through a displacement $\Delta x$ .




What happens to the speed of the object in this case?



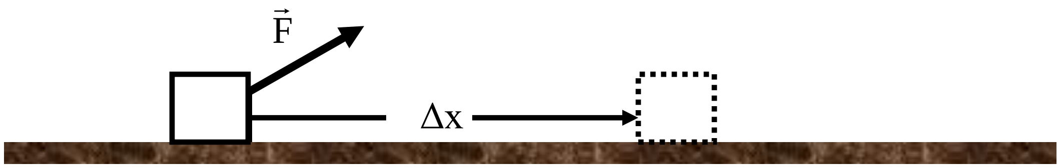

What happens to the speed of the block in this case?




What happens to the speed of the block in this case?

We see that the application of a force upon an object which under goes a displacement may affect the object's speed depending on how the force is orientated compared to the object's displacement, the size of the displacement, and the magnitude of the force. This leads us to the concept of work which is useful if finding the speeds of objects and for non-constant acceleration problems!!

## A. Definition of Work




The <u>infinitesimal</u> amount of **work** <u>done by</u> a **force**, $\vec{F}$, <u>upon</u> a **body** when <u>acting</u> <u>over</u> an **infinitesimal displacement**, $\Delta\vec{r}$, is given by

$$\Delta W \equiv \vec{F} \bullet \Delta\vec{r} = F\Delta x \,\mathrm{Cos}(\theta)$$

F is the magnitude of the force
$\Delta x$ is the displacement
$\theta$ is the angle between the force and displacement vectors

For our examples on the previous page, we have:

a) $\theta = 0°$, $W = F(\Delta x)$

b) $\theta = 90°$, $W = 0$

c) $\theta = 180°$, $W = -F(\Delta x)$.

Both force and displacement are vectors while work is a scalar. Thus we have a kind of multiplication of two vectors that produces a scalar instead of a vector. This type of multiplication of vectors called "scalar the product" or "dot product." You can find additional information about math tricks for doing this type of multiplication with Cartesian components and its many uses other than physics in math textbooks and the vector module.

**B.** The unit of work is the ___________________________________.

We **<u>don't</u>** use **<u>Nm</u>** because another quantity, TORQUE, also has these units.

**C. Evaluating Work**

To have work done on a body you must have:

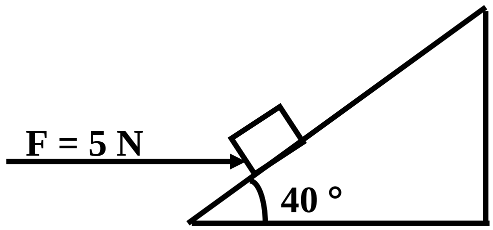
**1.** a force acting on the body.

**2.** a displacement.

**3.** a component of the force along the displacement.

**Note:** When calculating work, you must <u>first identify</u> the **work done** by **which <u>FORCE</u>**!! There may be several different forces with some or all of them doing work on the body.

**Example 1:** Calculate the work done by the 5.0 N force as the block is moved 5 m up an inclined plane with 40 degree inclination as shown below:




**Solution:**

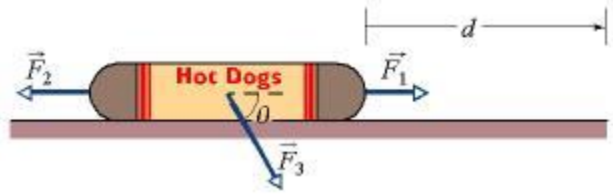
## D. Important Facts About Work

**1.** Work is a **<u>Scalar</u>**.

Thus, the **<u>net work</u>** on an object (ie the work by the **<u>net external force</u>**) is just

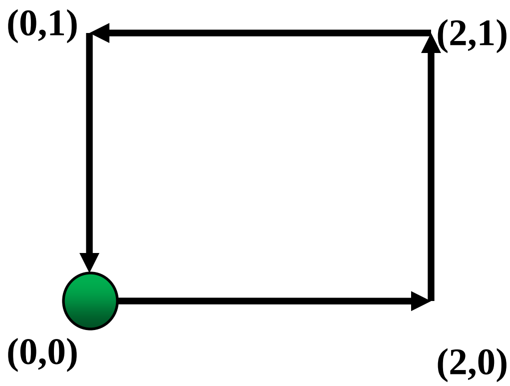
$$W_{NET} = W_1 + W_2 + W_3 + \ldots + W_N = \sum_{i=1}^{N} W_i$$

**Example:** The figure below shows a container of  hot dogs sliding rightward across a frictionless floor with three forces applied to it. Two of the forces are horizontal and have the magnitudes $F_1$ = 5.00 N and $F_2$ = 1.00 N; the third force is angled down by $\theta$ = 60.0° and has the magnitude $F_3$ = 4.00 N. If the hot dogs move through a distance $d$ = 0.35 m, find the work done by $F_1$, $F_2$, $F_3$, gravity, the normal force, and the net work.



**2.** In general, the amount of work done by a force on an object can change as the object moves along a path.

**Example:** Compute the work done by gravity on a 2.0 kg mass as it moves along the following paths:




**A)** From (0,0) to (2,0)

**B)** From (2, 0) to (2,1)

**C)** From (2,1) to (0,1)

**D)** From (0,1) to (0,0)

**E)** For the whole rectangular path

**3.** The work done by a force is **zero** if any of these four cases hold:

**a)** The **force** is **zero**.

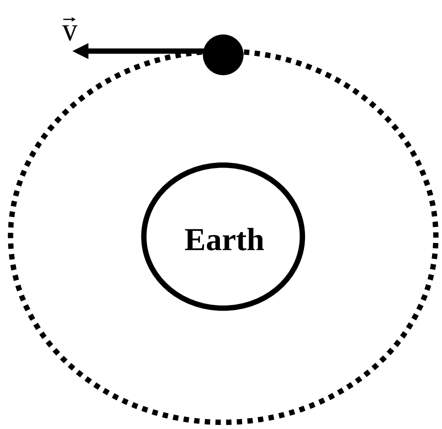
**b)** The **displacement** is **zero**.

**c)** The **force** is **perpindicular** to the **displacement**.

**d)** The **AREA under a force-distance graph** is **zero**.

As mentioned previously, unlike the force method, it is possible in the work approach to ignore a force if it does NO work!!

**Example:** The work done by Earth's gravity on the moon assuming that the moon travels in a circular orbit!!




The force is _______________

The displacement is ____________

The work is ______________________

Thus, the moon would **neither speed up nor slows down** as it goes around the Earth, but its **velocity vector changes direction so it is accelerating**!!

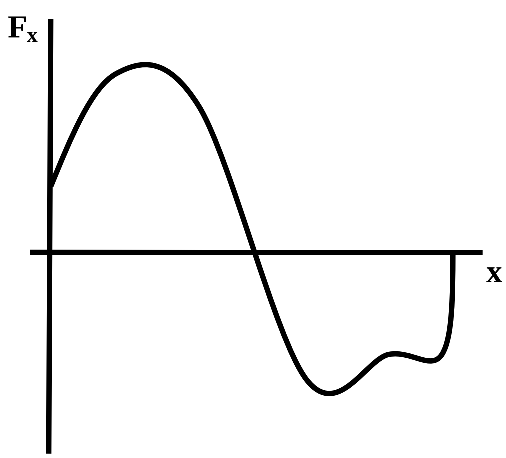
## D. Calculating Work For Variable Forces

**1.** The equation which I have given you for work is only valid as long as:
**a.** the magnitude of the force is constant
**b.** the angle between the force and the displacement vector is constant.

The ball example shows the basic idea for handling any work problem.

We break a path down into small segments over which both the magnitude of the force and the angle between the force and the displacement vectors are constant. For each segment, we calculate the work done by the force for that segement and add up all the works. If the segments are finite in size then we can do this by hand or with a spreadsheet. If the segments are infinitesimally small then we need more advanced math called Calculus.

**Calculus says that the work done on the object by a force is equal to the area under a graph of the force component vs displacement.**



**Example:** Calculate the work done by a force $\vec{F} = y\hat{\imath} + 3\hat{\jmath}$ on a particle as it moves from (0,0) to (1,2) by the following paths:

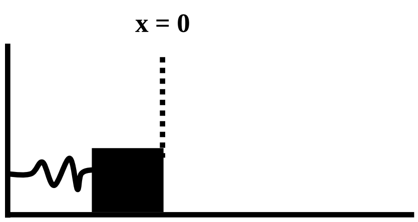
**a)** From (0,0) to (1,0) to (1,2)

**b)** From (0,0) to (0,2) to (1,2)

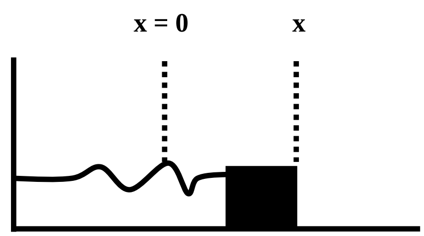
**Hooke Springs**

**A. Hooke's Law**

The magnitude of the restoring force on an ideal spring is **directly proportional** to the **amount** the spring is **stretched/compressed** from its un-stretched position. The direction of the spring force opposes the direction of **stretch** or **compression**.




**Un-stretched Position**




**Stretched Position**

**Hooke's Law:** $\vec{F} = -kx\hat{i}$

**The minus sign ensures that F is in the opposite direction of the displacement x.**

**Note:** This formula requires that x = 0 (origin) is placed at the unstretched position of the spring!!!

## B. Spring Constant

The spring constant contains **ALL** **information** about how the spring was **made**.

**Big k** means a **Stiff** spring.

**small k** means a **weak** spring.

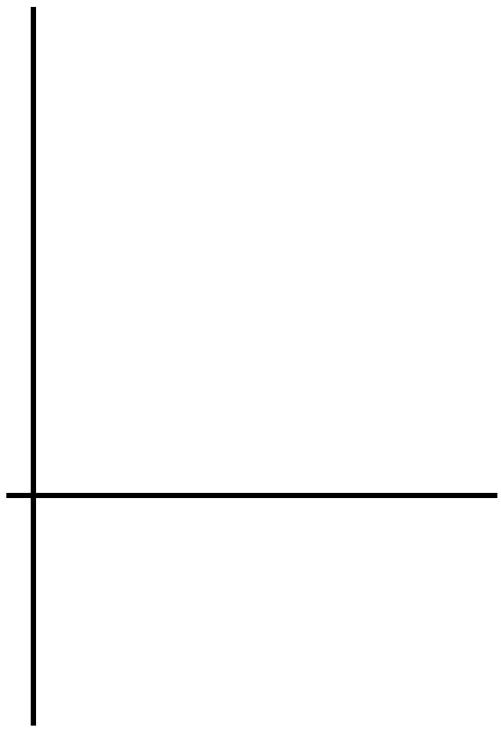
## C. Are real springs actually Hooke Springs?

**YES** if the **stretch** or **compression** is **really small**!

**Math Reason:** Any smooth function f (x) looks like a straight line if you don't move too far along the curve.

**D. Work Done By A Hooke Spring (Example of Calculating Work For Non- Constant Forces)**

**Graphically**



## III. Types of Work

We divide work into **two basic types** based upon whether the amount of work done **depends upon** the **path** that the object takes **or only** upon its **starting** and **ending point**!!

**A.** If the **Work** done by a force upon an object only depends on the object's **initial** and **final** location and **not** upon the **path** then the force is **conservative**.

**B.** All other forces are **non-conservative**.

**C.** **Unless you know that a force is conservative, you must treat it as a non-conservative force.**

**D.** The work done over a **closed** path by a **conservative** force is always **zero**.

**E.** **Examples of Conservative Forces: Gravity, Hooke Spring Force**