# Vectors & Scalars

**I.  Scalars**

**A.  Definition -**

A quantity that is **completely expressed** by a _____ and _____

**EXAMPLES:**

**B.**  Scalars may be either _____ or _____

**C.  Symbol -**

**D.  Scalar Field**   A function that associates at each location in space and time a specific scalar value.

An example of a scalar field is a map showing the temperature at various locations in the United States.

## II. Vectors

## A. Definition

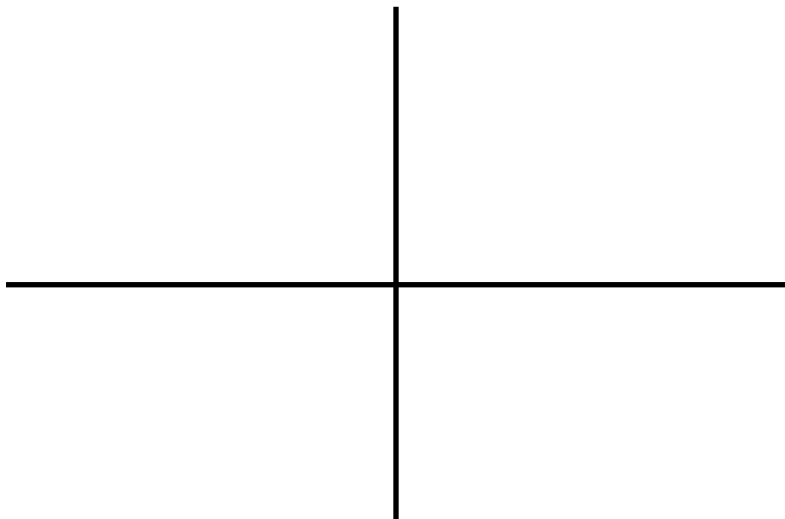A physical quantity that has both a _____ (       )

and a _____ .

## Examples:

## B. Symbol -

## C. Polar Representation

## 1. Graphically Description:

A vector can be described by _____ whose

_____ gives the _____ of

the vector and an _____ that gives the

_____.

**Note:** In advanced engineering and physics courses, we run into physical quantities that obey our definition but do not act like vectors. Thus, this definition although extremely useful is not always sufficient for advanced work. Instead, the vector like many mathematical entities is defined in terms of mathematical operations (transformations).

2. **Writing Vector in Polar Form:**

3. **Magnitude** (length) of the vector is a _____

_____.

It is the **hypotenuse** of our **triangle**!

4. **Most** measured **data** comes in this **form**, but it is **hard** to **make**

**calculations** in this **form**!!

**Example:  Graph the vector** $\bar{A}=6m/s\angle120°$

**5.**  **Unit Vectors -**  A **unit vector** is a vector with a **magnitude** of

_____ and has _____ **units**.

**Examples:**

**i)**  **Cartesian Unit Vectors**

**ii) Cylindrical Unit Vectors**

**iii) Spherical Unit Vectors**

## D. Graphical Addition of Vectors

We wish to add two vectors $\vec{A}$ and $\vec{B}$.

**Method:**

1. Draw vector $\vec{A}$ starting at the origin

2. Draw vector $\vec{B}$ starting with the tail of vector $\vec{B}$ at the tip of vector $\vec{A}$.

3. Draw the resultant vector ( $\vec{C}=\vec{A}+\vec{B}$ ) from the tail of $\vec{A}$ to the tip of $\vec{B}$.

4. Using a ruler and protractor measure the magnitude and direction of the resultant vector.

**EXAMPLE 1:** If $\vec{A}=5m\angle 60°$ and $\vec{B}=4m\angle 90°$ what is $\vec{C}=\vec{A}+\vec{B}$
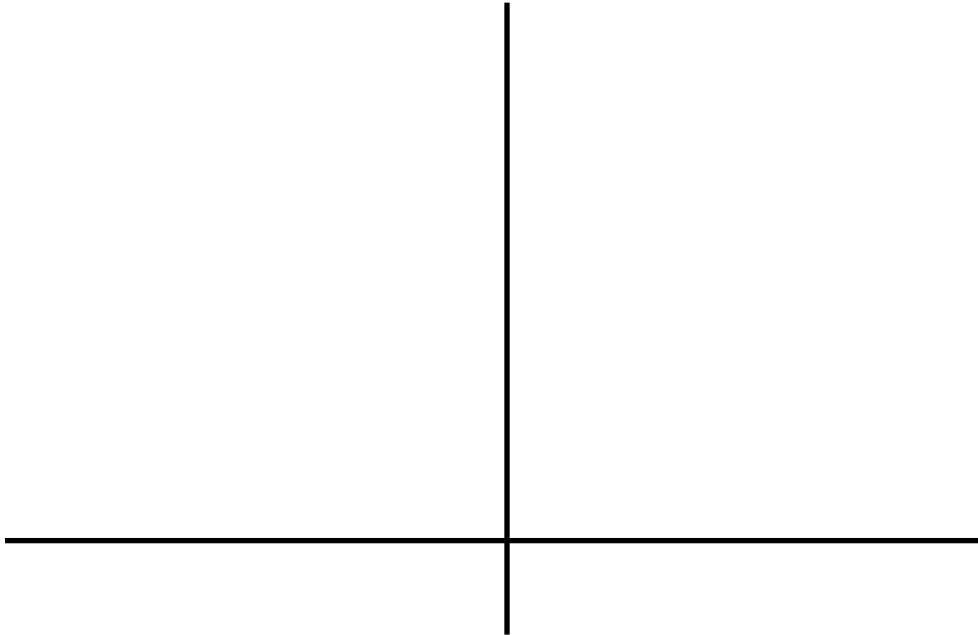
**Note:** Our graphical method of adding vectors proves that

$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

**E.  Multiplication of a Vector by a Scalar**

**1.**

From our knowledge of multiplying by scalars in elementary school, we

know that this is the same as adding the vector _____

_____ times.

**2.**  Thus, we see from our knowledge of vector addition that the

**magnitude** (**length**) of the original **vector** changes by the factor

_____ .

**Three Cases:**

   **i)**

   **ii)**

   **iii)**

**3.** If $k > 0$ then the direction of the resultant vector is the _____

as the original vector. If $k < 0$ then the direction of the resultant vector

is rotated _____ with respect to the original vector.

**Example:** Given that $\bar{A} = 2m\angle 30°$, find the following

**a)**        $5\bar{A}$        **b)** $0.1\bar{A}$        **c)** $-3\bar{A}$        **d)** $(2kg)\bar{A}$

## F.  Cartesian Representation

**1.**  The vector is broke into distances (components) along a set of unit vectors along the set of x, y, and z axis.

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**Example:**  For $\vec{B} = 3\hat{i} - 4\hat{j}$ do the following

**i)**  Draw the vector B

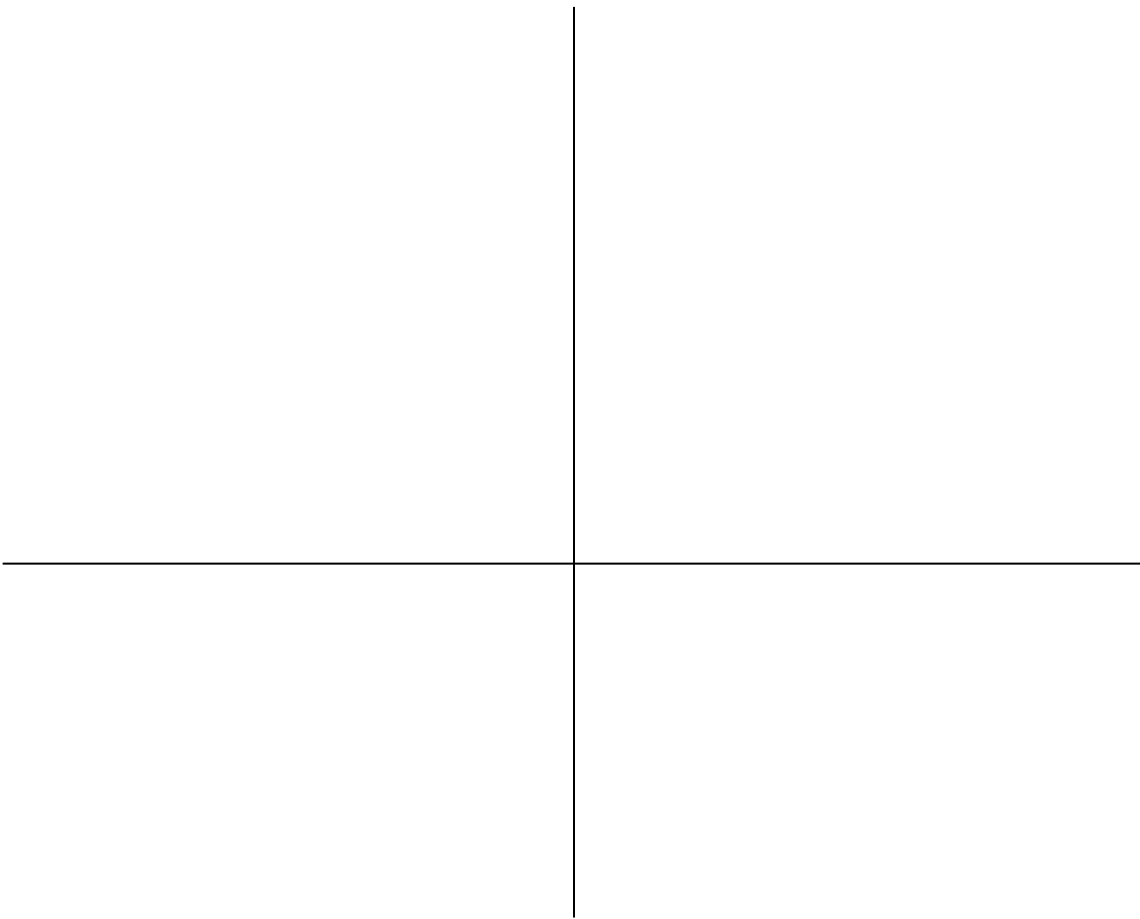**ii)**  What is the x-component?

**iii)**  What is the y-component?

Cartesian form is the easiest for doing calculations (adding and subtracting) vectors while Polar form is usually needed for presenting your work.

## 2. Adding Vectors in Cartesian Form

$$C_x = A_x + B_x$$

$$C_y = A_y + B_y$$

**Example:** Given $\vec{A}=3m\hat{i}-5m\hat{j}$ and $\vec{B}=4m\hat{i}+2m\hat{j}$, what is $\vec{A}+\vec{B}$ ?

## 3. Multiplying by a Scalar

Just **distribute** the **multiplication** of the **scalar** on to **each** of the vector's **components**.

**Examples:** Given the vector $\vec{A} = 3m\hat{i} - 4m\hat{j}$, write the following vectors in Cartesian form

a)                 $(3kg)\vec{A}$

b)                 $-2\vec{A}$

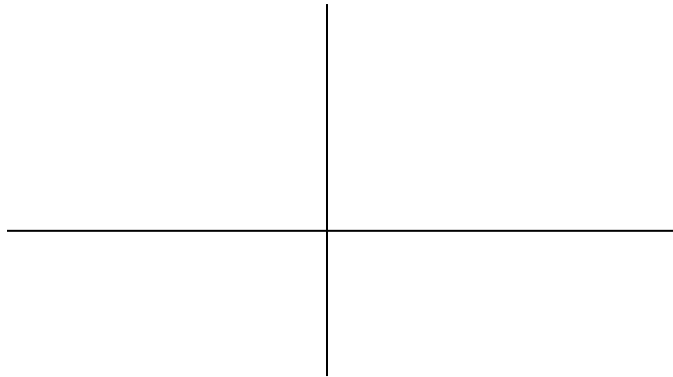### III. Polar to Cartesian Conversion

<p align="center"><b>B V I C</b></p>

**Method:**

1. Draw a vector sketch.
2. From the vector sketch, draw the associated triangle.
3. Find the lengths of the sides of the triangle.
4. Add signs to the values found in step #3 to get the vector components. (Signs depend on ± vector direction.
5. Check your work by drawing the result on the graph in #1. Do you get the same result?

**Example 1:** Consider the vector shown below
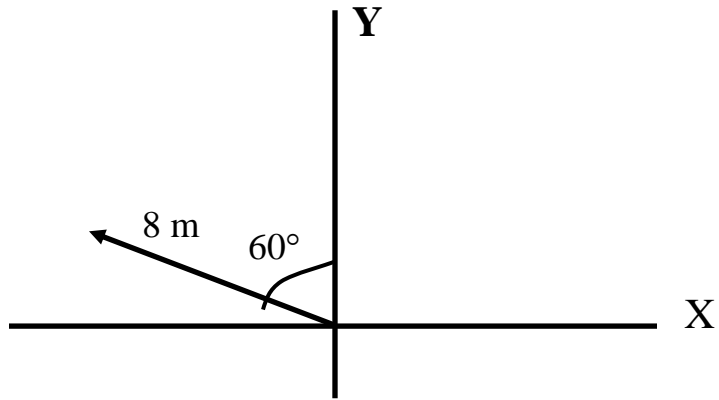
Using trigonometry and the enclosed triangles, we have

1)

2)

After determining the signs, we have the vector components

**Example 2:** Convert $\bar{A}=5\text{m}\angle 36.9°$ into Cartesian form.

**Example 3:** Write the vector shown below in Cartesian form.

## IV. Cartesian to Polar Representation

### Method:

1.  Draw a vector sketch.
2.  From the vector sketch, draw the associated triangle.
3.  Find the magnitude using the Pythagorean theorem

4.  Find the angle of the triangle using trigonometry (tangent function) and the triangle in step #2.
5.  You can get the final angle with respect to the +x-axis from the sketch in #1 using geometry. If you don't draw an angle on a sketch then I will assume that it is measured counter-clockwise from the +x-axis.

**Example:** Write $\vec{A} = -2\hat{i} - 5\hat{j}$ in Cartesian form.

1)

2)

3)

4)

5)

## V. Zero Vector

A **vector** is **<u>zero</u> if** and **only if** its **magnitude** (length) is _____

<u>**or equivalently**</u>

**<u>ALL</u>** of its _____ are _____.