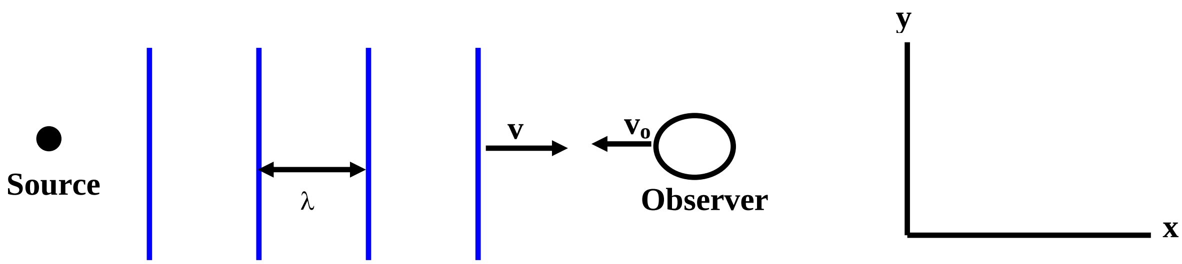
## Classical Doppler Shift

Anyone who has watched auto racing on TV is aware of the Doppler shift. As a race car approaches the camera, the sound of its engine increases in pitch (frequency) and decreases once the car passes the camera. We can use this pitch change to determine the relative speed of the car with respect to the air which carries the sound wave. This property is used in many real world applications including ultrasound imaging.

### A. Moving Observer

Assume that we have a stationary audio source that produces sound waves of frequency f and wave speed v. A stationary observer shown below sees the time between each wave as




$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$T = \frac{1}{f} = \frac{\lambda}{v}$$

If the observer is now moving at a velocity $v_o$ relative to the source then the speed of the waves as seen by the observer is

$$\text{Speed} = v \pm v_o$$

where the positive sign is when the observer is moving toward the source. The time between waves is now

$$T' = \frac{1}{f'} = \frac{\lambda}{v \pm v_o}.$$

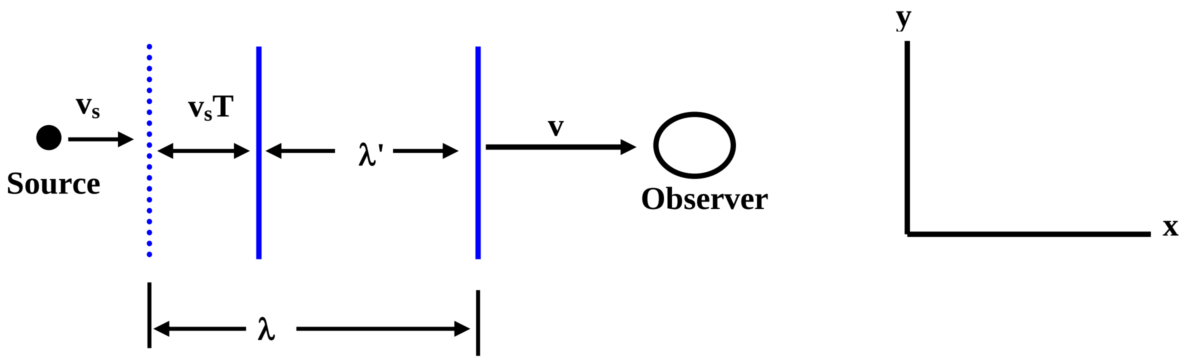
Taking the ratio of our two results we get that

$$\frac{T}{T'} = \frac{f'}{f} = \frac{v \pm v_o}{v}.$$

$$f' = f\left(\frac{v \pm v_o}{v}\right)$$

**B. Moving Source**

We now consider the case in which the source is moving toward the observer. In this case, the wave's speed is unchanged but the distance between wave fronts (wavelength) is reduced (increased) for the source moving toward (away) from the observer as shown below:




From the diagram, we find the new wavelength as

$$\lambda' = \lambda \mp v_s T$$

$$\frac{\lambda'}{\lambda} = \frac{\lambda \mp v_s T}{\lambda} = 1 \mp v_s \frac{T}{\lambda}$$

$$\frac{\lambda'}{\lambda} = 1 \mp \frac{v_s}{\lambda f}$$

$$\frac{\lambda'}{\lambda} = 1 \mp \frac{v_s}{v} = \frac{v \mp v_s}{v}$$

$$\frac{c}{\lambda}\frac{\lambda'}{c} = \frac{v \mp v_s}{v}$$

$$\frac{f}{f'} = \frac{v \mp v_s}{v}$$

Thus, the frequency seen by the observer for a moving source is given by

$$f' = f\left(\frac{v}{v \mp v_s}\right).$$

**Note:** The motion of the observer and source create different effects. For sound, this difference is explained due to motion relative to the preferred reference frame! This preferred frame is the reference frame stationary to the medium propagating the sound (air)!

### C. Combining Our Results

We can combine our two results to handle any situation as:

$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right)$$