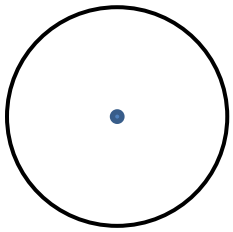
Energy Analysis of Rolling Without Slipping

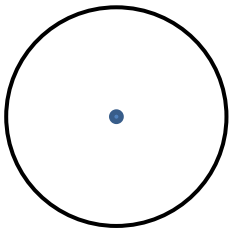
From Chassel's Theorem, we know that the total kinetic energy has two parts (Rotation and Translation).

$$K_{Total} = \frac{1}{2} M {V_{cm}}^2 + \frac{1}{2} I_{cm} \omega^2$$

For an object undergoing general motion, the speed of the center of mass and the angular speed about the center of mass are independent.

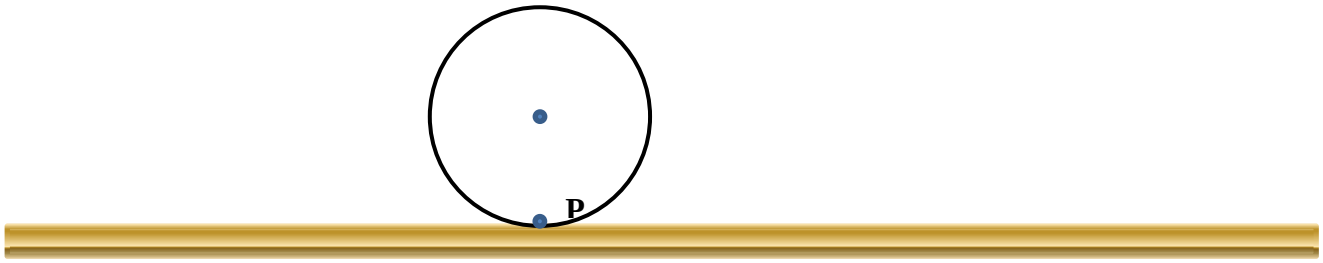
Examples:





A special case where the speed of the center of mass and the angular speed about the center of mass are not independent is rolling without slipping. Here we have the no slip condition:

$$V_{cm} = \omega R$$



Substituting the no-slip condition into our energy equation removes one variable. We can write the total kinetic energy either as:

$$K_{Total} = \frac{1}{2} M {V_{cm}}^2 + \frac{1}{2}\frac{I_{cm}}{R^2}{V_{cm}}^2$$

Or

$$K_{Total} = \frac{1}{2} M R^2\omega^2 + \frac{1}{2} I_{cm}\omega^2$$