The Dot Product (Scalar Vector Multiplication)

V. Scalar Vector Multiplication

1. Notation

2. Result is a **scalar** whose **value** is found by the formula

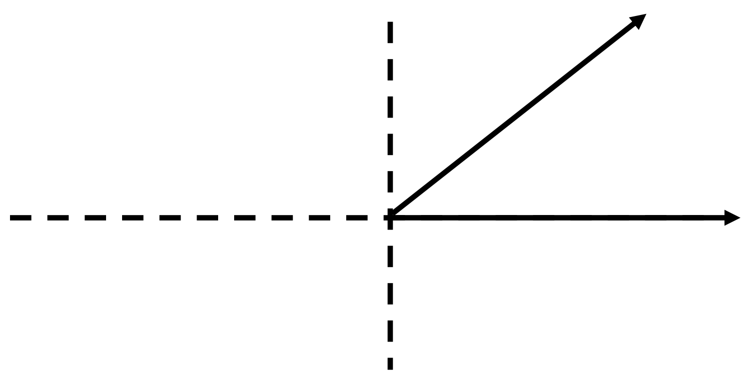
3. Graphical Interpretation

From the diagrams, we see that the **dot product** of two vectors, A and B, can be viewed either as the **length of vector A** <u>times</u> the <u>component</u> of <u>vector B</u> **along <u>vector A's direction</u>**.

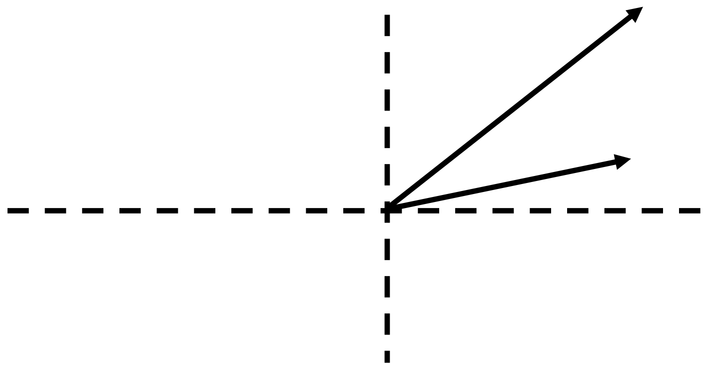
or equivalently

the **length of vector B** <u>times</u> the <u>component</u> of <u>vector A</u> **<u>along vector B</u>**.

Example 1: Find the dot product of the two vectors shown below:



Example 2: Find the dot product of the two vectors shown below:



4. Cartesian Form of the Dot Product

Our existing form for the dot product is very inconvenient for dealing with vectors that already in Cartesian form. We can obtain a useful form for dealing with vectors in Cartesian form **simply <u>multiplying the unit vectors</u>** like **<u>ordinary</u> variables** in **Algebra** and then <u>using</u> our <u>existing</u> **dot product formula** to **<u>evaluate</u>** the **results**.

Given $\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$ and $\vec{B} = B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$, the dot product of the two vectors is

Proof:

Example: Given $\vec{A} = 3\hat{i} + 2\hat{j}$ and $\vec{B} = -4\hat{i} + 3\hat{j}$, find the dot product.

5. Application of the Dot Product

The dot product has many important applications. In fact, you have already seen one result of the dot product (Pythagorean Theorem)!!

a) Calculating A Vector's Magnitude

Example: What is the magnitude of the vector: $\vec{B} = -2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

b) Creating A Unit Vector

If you need to create a unit vector that points in the same direction as another vector $\vec{A}$, then you can do the following:

Example: Find a unit vector pointing along the direction of the vector: $\vec{A} = 3\hat{\imath} + 2\,\hat{\jmath} - 4\,\hat{k}$

c) Finding the Component of a vector

Physicists and engineers often need to **express vectors** in **coordinate systems** other than just Cartesian. We can **determine the components** of the **vector** along any **set of unit vectors** using the **dot product**.

Example: Directional Cosines

Using the Cartesian unit vectors, we can develop the directional cosines method of expressing a three dimensional vector. This for is very useful for solving engineering problems (used in Engineering Principles I).