

## Fluids

### I. Continuity Equation

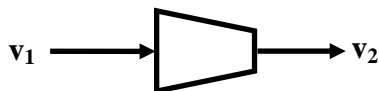
For an incompressible fluid the volume of fluid passing through any size cross section of a pipe must be the same due to conservation of mass. This condition can also be stated in a more useful form as follows:

The product of the cross section and the component of the velocity of the fluid perpendicular to the cross sectional area is a constant.

$$\text{Flux} =$$

This conserved quantity is called flux which is Latin for “water flow.”

**Proof:** Consider a fluid flowing through a pipe of varying cross sectional area as shown below.



An amount of fluid of mass  $M$  enters the pipe on the left at speed  $v_1$ . Since the amount of fluid in the pipe can not increase, the same mass of fluid must be pushed out on the right side of the pipe at speed  $v_2$ . We can write the mass of the volume of fluid entering the left hand side over a time  $t$  as

We can write the mass of the volume of fluid exiting the right hand side over a time  $t$  as

Equating our two results gives us the continuity equation as

The continuity equation explains many interesting everyday events including the fact that when water flows through a reduced cross sectional area it will travel at a higher speed. This is why we reduce the area of a garden hose to make a sprayer and why fast moving rapids occur when a river becomes shallow!!! It is just a simple consequence of conservation of atoms!!!