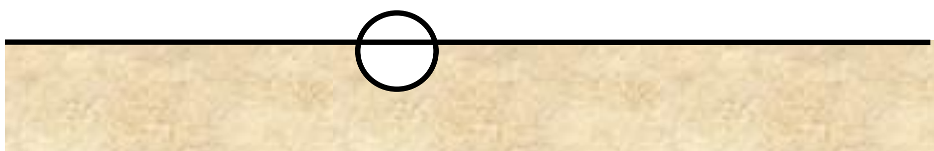
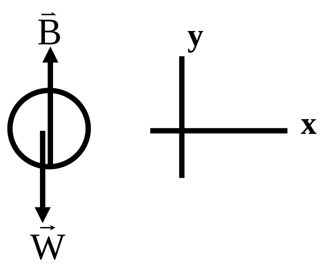
Fluids

I. Archimedes's Principle

> Any object completely or partially submerged in a fluid is buoyed up by a force with magnitude equal to the weight of the fluid displaced by the object.

Consider an object of mass m_o that is partially or totally submerged like the one shown below with its free body diagram.





Using Newton's Second Law, we have

The weight of the object can be related to the density of the object, ρ_o, and the object's total volume V by

Using Archimedes's principle, the buoyancy force is related to density of the fluid ρ_f, and the volume of the fluid displaced (this is the portion of the volume of the object which is submerged) V_s.

B =

Substituting into our Newton II work we have

Replacing m_o in terms of the object's density, we have

Case I: Floating (Partially Submerged) Object

In this case the object is in equilibrium so $A_y = 0$. This gives us the following condition:

$$\frac{V_s}{V} = \left(\frac{\rho_o}{\rho_f}\right) \leq 1$$

This says that the fraction of the volume of the object that is submerged is equal to the ratio of the density of the object to the density of the fluid. Since the largest fraction of the volume that can be submerged is 100%, we also see that the density of the object must be less than the density of the fluid if the object is to float.

Case II: Totally Submerged Object

In this case the volume of water displaced is equal to the volume of the object.

$$V_s = V$$

Putting this condition in our Newton II results, we have

If the object's density if greater than the fluid then the object will accelerate downward (sink) until it hits the bottom and an additional force is added to our free body diagram. If the object's density is less than the density of the fluid, the object will accelerate upward to the surface and will become only partially submerged as in case II. If the density of the object and the fluid is the same then the object will be in equilibrium and remain submerged at this depth.