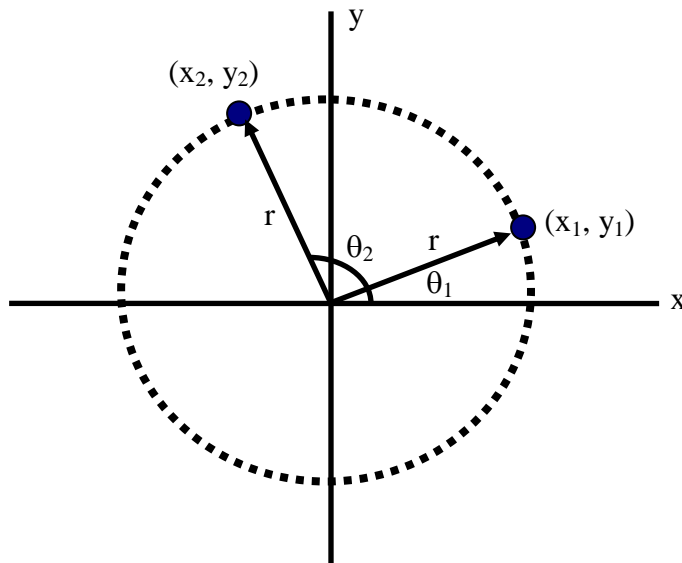# Rotation Basics

I.   Angular Position
A.   Background

Consider a student who is riding on a merry-go-round. We can represent the student's location by using either Cartesian coordinates or by using cylindrical coordinates as shown below.


In Cartesian coordinates system, both of the student's position coordinates change over time. However, if we use cylindrical coordinates, only the student's angular coordinate, θ, changes over time. This may not seem like much of an advantage, but it is actually a huge savings when you consider that the change in the angle of every single point on the merry-go-around is the same!!! This means that we can replace solving for 10^{24} or more coordinates for the particles of the merry-go-round or any rigid rotating object with a single coordinate, θ, in a manner similar to the way that we use the center-of-mass.

B.   Symbol –

C.   Relationship to Distance Traveled

Consider the two runners below who make a complete lap around a circular track. Although every runner covers an angular displacement of 2π radians, the runner whose circle has the greatest radius travels a greater distance!!
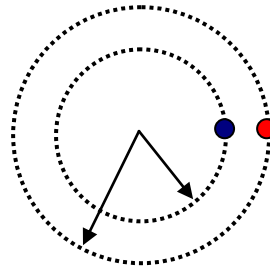
The relationship between the arc length (distance covered) and the angle of the arc (our angular position) is

$$S =$$

This relationship actually defines the angular unit called the radian!!!

D.   Units –

The **RADIAN** is a **unitless quantity**. We retain it in the units of other rotational quantities (angular speed, etc.) even though it is unitless to remind us that we are dealing with an angular quantity.

It is important to realize that although angular position can be specified using units other than radians including degrees and revolutions, you must convert to radians before relating the

angular part of a problem to the distance traveled or other non-angular part.

**E.   Angular Conversions**

_____ Radians = _____ Revolution

_____ Radians = _____ Degrees

EXAMPLE: The runners covered an angular displacement of 200 degrees while running on a circular track of radius 150 m.

F.   Angular position is NOT a vector!!!

If we rotate a textbook about two different axis, we can show that the result of the two rotations depends on the order in which the rotations were performed. Thus, angular position doesn't obey the associative law of vector addition.

$$\theta_1 + \theta_2 \neq \theta_2 + \theta_1$$
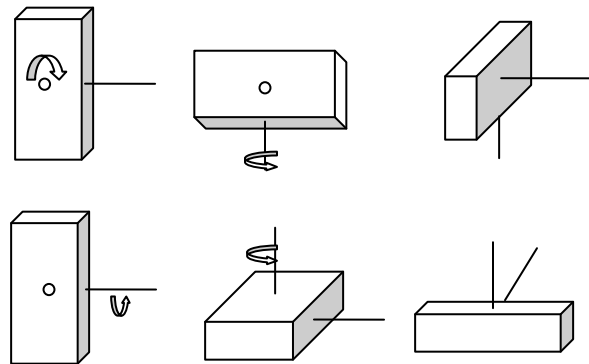
This may be your first introduction to objects with magnitude and direction that are not vectors!! It will probably not be your last. This is why advanced math classes define vector more rigorously in terms of mathematical operations and not vague terms like size and directions.

G.   An infinitesimal change in the angular position, $\Delta\vec{\theta}$, is a vector!!!

As we reduce the size of the angles through which we rotate the books, the difference between the two rotation sequences approaches zero!!

II. Angular Velocity - $\bar{\omega}$

A. Definition – The angular velocity is defined as the

_____   _____ of _____ of

_____ _____ .

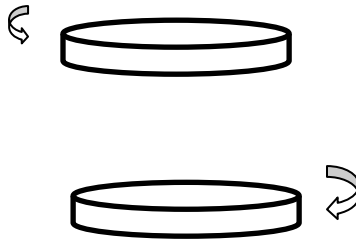| |
|---|
| |

B. Units - _____

C. Relationship to Tangential Speed
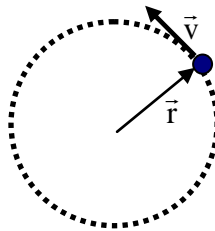

D. Angular velocity is a _____.

E. The direction of the angular velocity is determined by the

_____ _____ Rule.

Right Hand Rule for Circular Motion

_____ the _____ of your right hand in the

_____ of _____, your

_____ points in the direction of $\vec{\omega}$.

It is important to understand that we do this because everyday conventions like "clockwise" and "counter-clockwise" are ambiguous for 3-D problems since they depend on the position of the observer.

III. Angular Acceleration - $\vec{\alpha}$

A. Definition – The angular acceleration is defined as the

_____ _____ of

_____ of _____ _____.

$$\begin{array}{|c|}\hline \\ \\ \\ \\ \hline \end{array}$$

B. Units - _____

C. Relationship to Tangential Acceleration

$$\begin{array}{|c|}\hline \\ \\ \\ \\ \hline \end{array}$$

D. Angular acceleration is a _____.

E. Direction is determined by determining if the angular velocity is increasing or decreasing:

Angular speed is increasing then the angular acceleration

points in the _____ _____ as the

_____ _____.

Angular speed is decreasing then the angular acceleration

points in the _____ _____ as the

_____ _____.

Increasing Angular Speed

Decreasing Angular Speed

F.   **Important Past Information**

All objects **undergoing rotation** have **centripetal acceleration** even if they have no tangential acceleration!!

A useful formula relating the magnitude of the centripetal acceleration to the angular speed is

The total acceleration is always the vector sum of the tangential acceleration and the centripetal acceleration:

IV. Big Board and Developing Rotation Equations

Because we started all of our work on rotations by simply changing our coordinate system, we shouldn't be surprised that there are direct connections between our past equations and new equations in rotation land. The physics would appear to be the same. In fact, some of the concepts like angular momentum turn out to be deeper than just a different way to state old physics. They turn out to be true at the atomic world even when our classical analogies break down!!

A. Big Board Trick for Rigid Body Rotation About A Fixed Axis

You can develop rotational formulas by using linear formulas that we previous studied and replacing the linear variables with their rotational analogs from the following Big Board.

Property	Linear Variable	Rotational Variable
Position		
Velocity		
Acceleration		
Inertia		
Cause of Acceleration		
Momentum		
Energy		
Time		

Example: Develop the rotational kinematic equations and their conditions for validity.

Example: Write Newton's 2nd Law for a rotational problem with constant rotational inertia.

Example: Write the formula for calculating rotational kinetic energy if the object has constant rotational inertia.

V. Rotational Kinematics

A. These problems are identical to 1-D linear kinematic problems like those that you worked in Chapter 2. It is important to remember that
these equations can be used only in problems in which the **angular acceleration** is **constant** or in which you can **break the problem** into a series of connected smaller problems in which the **angular acceleration** is **constant** in each of the smaller problems.

B. Rotational graphs can also be used to solve rotational kinematic problems in the same manner that you used graphs in Chapter 2 for 1-D linear problems.

Example: A wheel is rotating at constant angular speed for four seconds before it trips a breaker and then de-accelerates at a constant rate till it stops 6 seconds later. If the wheel rotates through a total angle of 300 rad, what was the initial angular speed of the wheel?