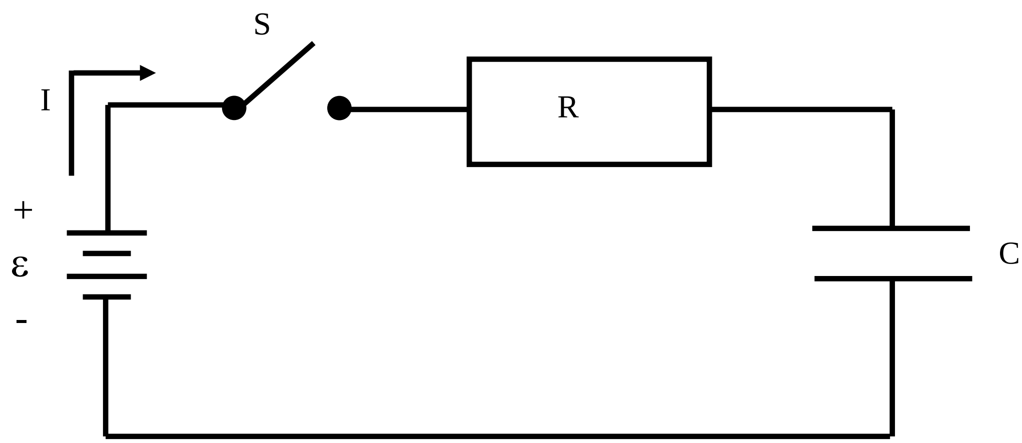
RC Circuits

I. Charging A Capacitor

A. Before Switch Is Closed: t < 0 s

1. The Circuit



2. Current Flowing In Circuit – I_{o-}

3. Voltage Across The Resistor – V_{R0-}

Using Ohm's law, we have that

4. Charge On The Plates Of The Capacitor – Q_{0-}

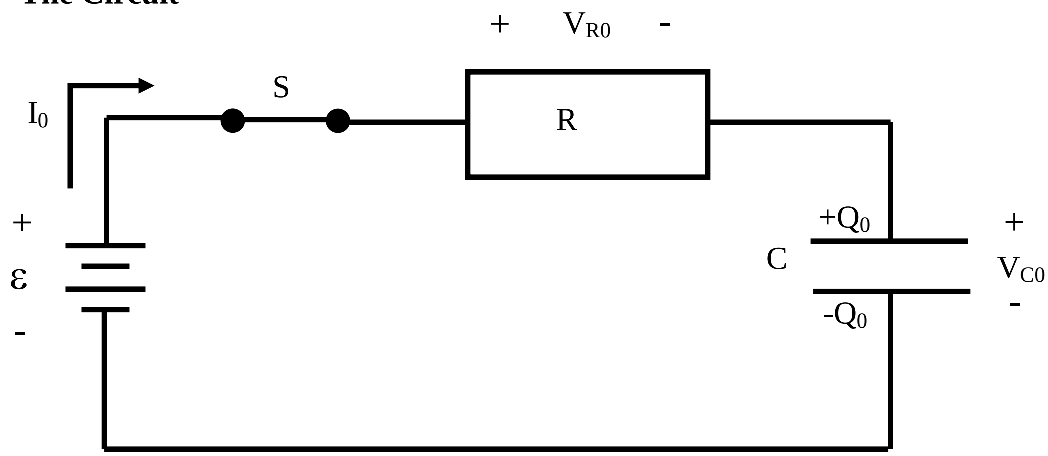
In our problem, we will assume that no charge has been left on the capacitor's plate from a previous charging event.

5. Voltage Across The Capacitor – V_{c0-}

Using the definition of capacitance, we have

B. Immediately After Closing The Switch: t = 0 s

1. The Circuit



2. Charge On The Plate's Of The Capacitor's – Q_0

Because charged particles like electrons do not travel at infinite speeds, it will take at least a small finite amount of time for these particles to travel through the circuit and accumulate/leave the plates of the capacitor.

Thus, we have the following important condition for any RC circuit

> The charge on a capacitor cannot change instantaneously (i.e. the function must be continuous).

In our particular problem, this corresponds to

3. Voltage Across The Capacitor – V_{C0}

Since the voltage across a capacitor is directly proportional to the charge on the plates of the capacitor, our previous work can be restated as follows.

> The voltage across a capacitor can not change instantaneously (i.e. the function must be continuous).

Thus, we have

4. **Voltage Across The Resistor – V_{R0}**

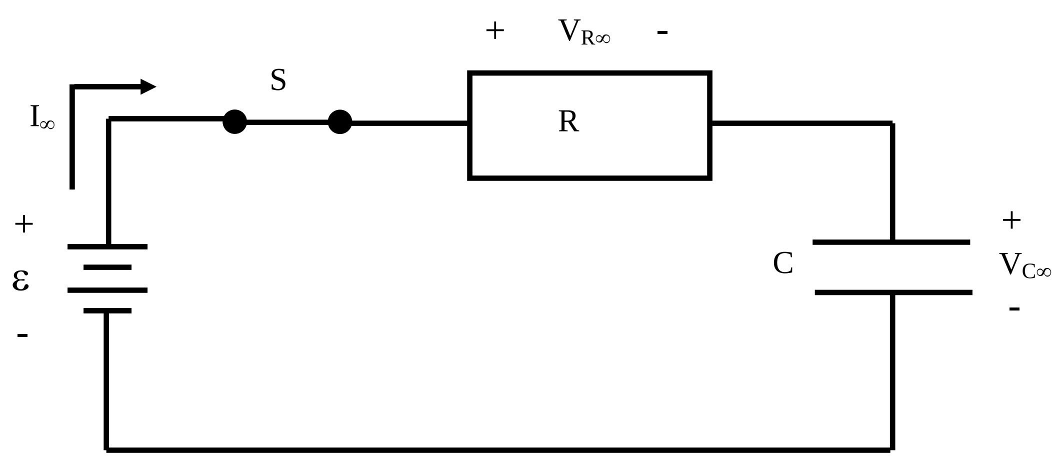
Since our capacitor initially acts as a short circuit, we have by Kirchhoff's voltage law that

5. **Current Flowing In The Circuit – I_0**

Using Ohm's Law, we have

C. **After Switch Has Been Closed For A Long Time: t → ∞ s**

1. **The Circuit**



2. **Current Flowing In The Circuit - I_∞**

As charge increases on the capacitor plates, it becomes harder for new charge carriers to move between the plates. Eventually, the current flowing through the circuit will become **ZERO**.

Thus, we have the following important relationship:

a charging capacitor will eventually become an **OPEN** circuit.

3. **Voltage Across Resistor - $V_{R\infty}$**

Using Ohm's Law, we have

4. **Voltage Across The Capacitor - $V_{C\infty}$**

By Kirchhoff's Voltage Law, we have

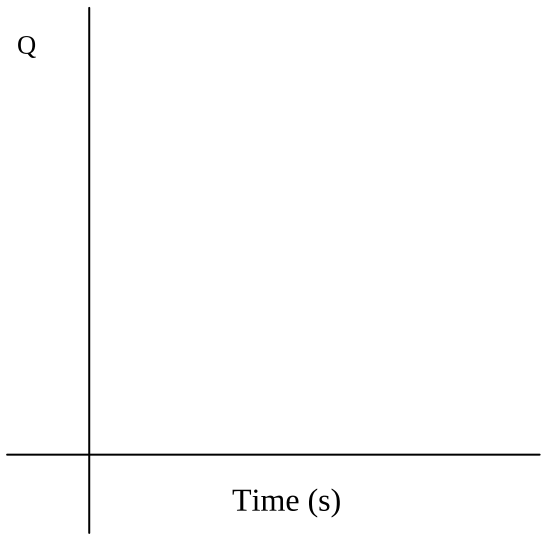
5. **Charge On The Plates Of The Capacitor - Q_∞**

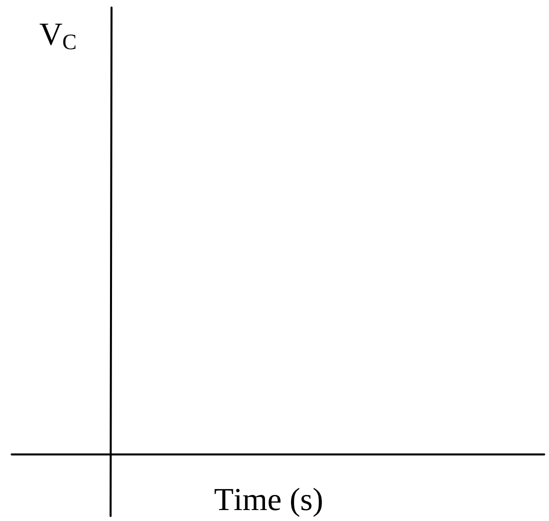
From the definition of capacitance, we know that

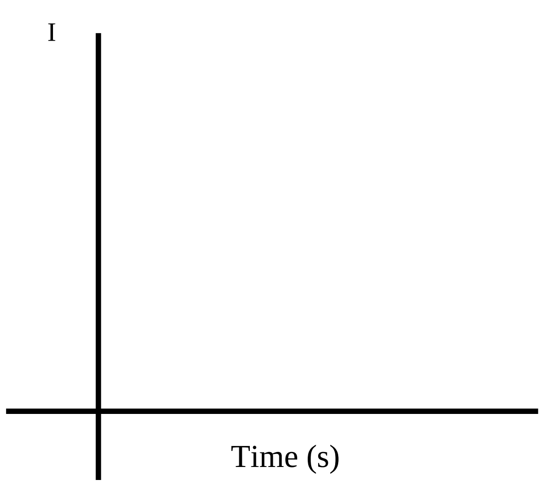
D. **Graphical Results For A Charging Capacitor**

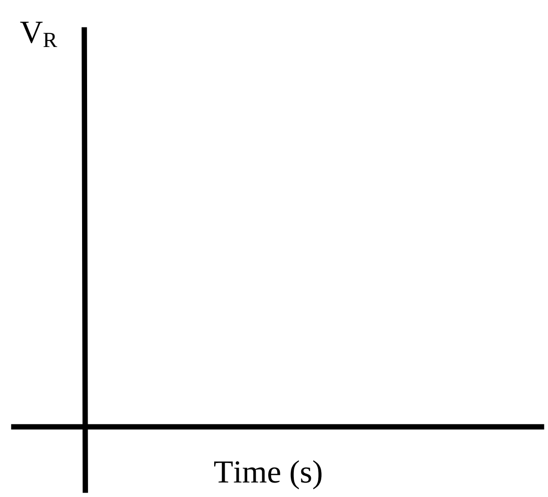
1. We plot our three points for each graph that we found in parts A, B, and C.

2. On each graph, we can connect our points from parts B and C by realizing that the graphs must be continuos and that they are limited by the values found in part C.









$$I(t) = I_0 \, e^{-\left(\frac{t}{\tau}\right)} = \frac{\varepsilon}{R} e^{-\left(\frac{t}{\tau}\right)}$$

$$V_R(t) = V_{R\,max} \, e^{-\left(\frac{t}{\tau}\right)} = \varepsilon \; e^{-\left(\frac{t}{\tau}\right)}$$

$$Q(t) = Q_{max}\left(1 - e^{-\left(\frac{t}{\tau}\right)}\right) = \varepsilon C \left(1 - e^{-\left(\frac{t}{\tau}\right)}\right)$$

$$V_C(t) = V_{C\,max}\left(1 - e^{-\left(\frac{t}{\tau}\right)}\right) = \varepsilon \left(1 - e^{-\left(\frac{t}{\tau}\right)}\right)$$

E. RC Time Constant

The time it takes to charge a capacitor depends on the resistance and capacitance of the circuit.