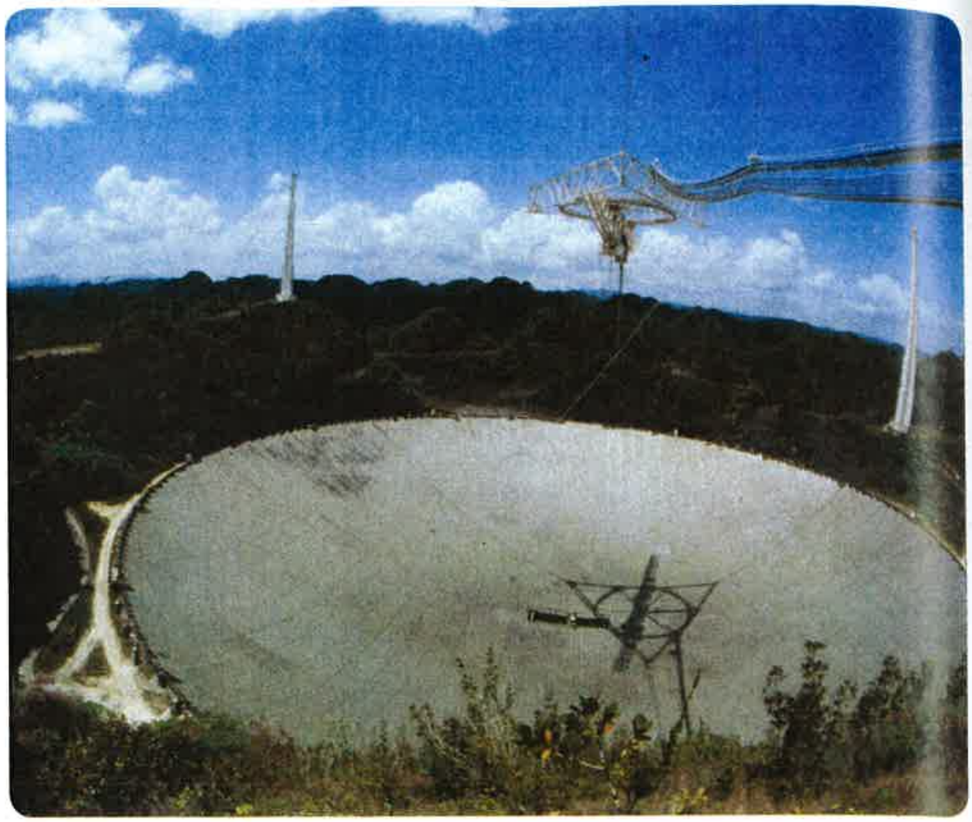
21

Arecibo, a large radio telescope in Puerto Rico, gathers electromagnetic radiation in the form of radio waves. These long wavelengths pass through obscuring dust clouds, allowing astronomers to create images of the core region of the Milky Way galaxy, which can't be observed in the visible spectrum.



ALTERNATING-CURRENT CIRCUITS AND ELECTROMAGNETIC WAVES

Every time we turn on a television set, a stereo system, or any other electric appliances, we call on alternating currents (AC) to provide the power to operate them. We begin our study of AC circuits by examining the characteristics of a circuit containing a source of emf and one other circuit element: a resistor, a capacitor, or an inductor. Then we examine what happens when these elements are connected in combination with each other. Our discussion is limited to simple series configurations of the three kinds of elements.

We conclude this chapter with a discussion of **electromagnetic waves,** which are composed of fluctuating electric and magnetic fields. Electromagnetic waves in the form of visible light enable us to view the world around us; infrared waves warm our environment; radio-frequency waves carry our television and radio programs, as well as information about processes in the core of our galaxy; and X-rays allow us to perceive structures hidden inside our bodies and study properties of distant, collapsed stars. Light is key to our understanding of the universe.

21.1 RESISTORS IN AN AC CIRCUIT

An AC circuit consists of combinations of circuit elements and an AC generator or an AC source, which provides the alternating current. We have seen that the output of an AC generator is sinusoidal and varies with time according to

$$\Delta v = \Delta V_{\text{max}} \sin 2\pi f t \qquad [21.1]$$

where Δv is the instantaneous voltage, ΔV_{max} is the maximum voltage of the AC generator, and f is the frequency at which the voltage changes, measured in hertz (Hz). (Compare Equations 20.7 and 20.8 with Equation 21.1.) We first consider a simple circuit consisting of a resistor and an AC source (designated by the symbol ), as in Active Figure 21.1. The current and the voltage versus time across the resistor are shown in Active Figure 21.2.

To explain the concept of alternating current, we begin by discussing the current versus time curve in Active Figure 21.2. At point *a* on the curve, the current has a maximum value in one direction, arbitrarily called the positive direction. Between points *a* and *b*, the current is decreasing in magnitude but is still in the positive direction. At point *b*, the current is momentarily zero; it then begins to increase in the opposite (negative) direction between points *b* and *c*. At point *c*, the current has reached its maximum value in the negative direction.

The current and voltage are in step with each other because they vary identically with time. **Because the current and the voltage reach their maximum values at the same time, they are said to be in phase.** Notice that **the average value of the current over one cycle is zero** because the current is maintained in one direction (the positive direction) for the same amount of time and at the same magnitude as it is in the opposite direction (the negative direction). The direction of the current, however, has no effect on the behavior of the resistor in the circuit: the collisions between electrons and the fixed atoms of the resistor result in an increase in the resistor's temperature regardless of the direction of the current.

We can quantify this discussion by recalling that the rate at which electrical energy is dissipated in a resistor, the power $\mathcal{P}$, is

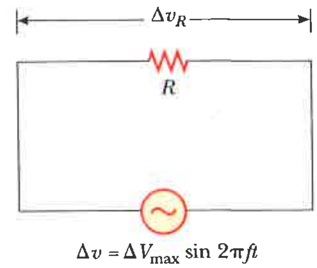
$$\mathcal{P} = i^2 R$$

where i is the *instantaneous* current in the resistor. Because the heating effect of a current is proportional to the *square* of the current, it makes no difference whether the sign associated with the current is positive or negative. The heating effect produced by an alternating current with a maximum value of I_{max} *is not the same* as that produced by a direct current of the same value, however. The reason is that the alternating current has this maximum value for only an instant of time during a cycle. The important quantity in an AC circuit is a special kind of average value of current, called the **rms current**: the direct current that dissipates the same amount of energy in a resistor that is dissipated by the actual alternating current. To find the rms current, we first square the current, then find its average value, and finally take the square root of this average value. Hence, the rms current is the square *root* of the average (*mean*) of the *square* of the current. Because i^2 varies as $\sin^2 2\pi ft$, the average value of i^2 is $\frac{1}{2}I_{max}^2$ (Fig. 21.3b, page 698).[1] Therefore, the rms current I_{rms} is related to the maximum value of the alternating current I_{max} by

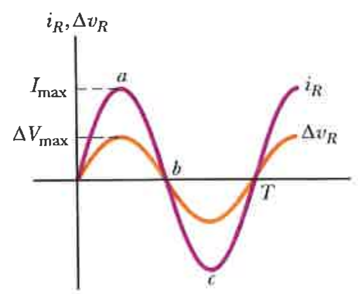
$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max} \qquad [21.2]$$

This equation says that an alternating current with a maximum value of 3 A produces the same heating effect in a resistor as a direct current of $(3/\sqrt{2})$ A. We can therefore say that the average power dissipated in a resistor that carries alternating current I is

$$\mathcal{P}_{av} = I_{rms}^2 R$$



ACTIVE FIGURE 21.1
A series circuit consisting of a resistor R connected to an AC generator, designated by the symbol



ACTIVE FIGURE 21.2
A plot of current and voltage across a resistor versus time.

[1]We can show that $(i^2)_{av} = I_{max}^2/2$ as follows: The current in the circuit varies with time according to the expression $i = I_{max} \sin 2\pi ft$, so $i^2 = I_{max}^2 \sin^2 2\pi ft$. Therefore, we can find the average value of i^2 by calculating the average value of $\sin^2 2\pi ft$. Note that a graph of $\cos^2 2\pi ft$ versus time is identical to a graph of $\sin^2 2\pi ft$ versus time, except that the points are shifted on the time axis. Thus, the time average of $\sin^2 2\pi ft$ is equal to the time average of $\cos^2 2\pi ft$, taken over one or more cycles. That is,

$$(\sin^2 2\pi ft)_{av} = (\cos^2 2\pi ft)_{av}$$

With this fact and the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$, we get

$$(\sin^2 2\pi ft)_{av} + (\cos^2 2\pi ft)_{av} = 2(\sin^2 2\pi ft)_{av} = 1$$

$$(\sin^2 2\pi ft)_{av} = \tfrac{1}{2}$$

When this result is substituted into the expression $i^2 = I_{max}^2 \sin^2 2\pi ft$, we get $(i^2)_{av} = I_{rms}^2 = I_{max}^2/2$, or $I_{rms} = I_{max}/\sqrt{2}$, where I_{rms} is the rms current.

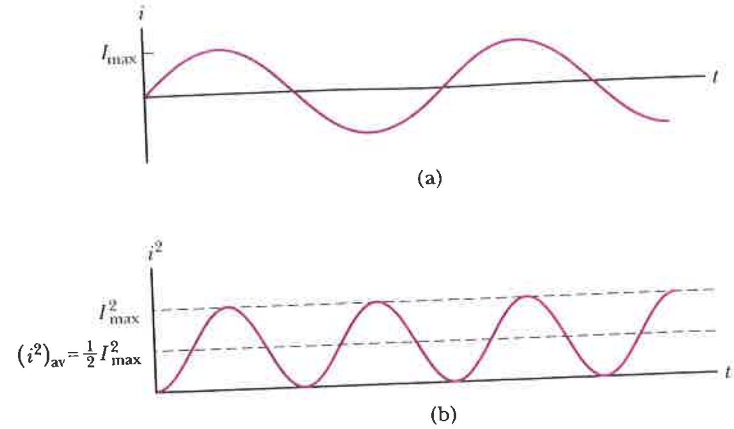


FIGURE 21.3 (a) Plot of the current in a resistor as a function of time. (b) Plot of the square of the current in a resistor as a function of time. Notice that the gray shaded regions *under* the curve and *above* the dashed line for $I^2_{max}/2$ have the same area as the gray shaded regions *above* the curve and *below* the dashed line for $I^2_{max}/2$. Thus, the average value of I^2 is $I^2_{max}/2$.

Alternating voltages are also best discussed in terms of rms voltages, with a relationship identical to the preceding one,

rms voltage →

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\, \Delta V_{max} \qquad [21.3]$$

where ΔV_{rms} is the rms voltage and ΔV_{max} is the maximum value of the alternating voltage.

When we speak of measuring an AC voltage of 120 V from an electric outlet, we actually mean an rms voltage of 120 V. A quick calculation using Equation 21.3 shows that such an AC voltage actually has a peak value of about 170 V. In this chapter we use rms values when discussing alternating currents and voltages. One reason is that AC ammeters and voltmeters are designed to read rms values. Further, if we use rms values, many of the equations for alternating current will have the same form as those used in the study of direct-current (DC) circuits. Table 21.1 summarizes the notations used throughout this chapter.

TABLE 21.1

Notation Used in This Chapter

	Voltage	Current
Instantaneous value	Δv	i
Maximum value	ΔV_{max}	I_{max}
rms value	ΔV_{rms}	I_{rms}

Consider the series circuit in Figure 21.1, consisting of a resistor connected to an AC generator. A resistor impedes the current in an AC circuit, just as it does in a DC circuit. Ohm's law is therefore valid for an AC circuit, and we have

$$\Delta V_{R,rms} = I_{rms} R \qquad [21.4a]$$

The rms voltage across a resistor is equal to the rms current in the circuit times the resistance. This equation is also true if maximum values of current and voltage are used:

$$\Delta V_{R,max} = I_{max} R \qquad [21.4b]$$

QUICK QUIZ 21.1 Which of the following statements can be true for a resistor connected in a simple series circuit to an operating AC generator? (a) $\mathcal{P}_{av} = 0$ and $i_{av} = 0$ (b) $\mathcal{P}_{av} = 0$ and $i_{av} > 0$ (c) $\mathcal{P}_{av} > 0$ and $i_{av} = 0$ (d) $\mathcal{P}_{av} > 0$ and $i_{av} > 0$

EXAMPLE 21.1 What Is the rms Current?

Goal Perform basic AC circuit calculations for a purely resistive circuit.

Problem An AC voltage source has an output of $\Delta v = (2.00 \times 10^2 \text{ V}) \sin 2\pi ft$. This source is connected to a $1.00 \times 10^2\ \Omega$ resistor as in Figure 21.1. Find the rms voltage and rms current in the resistor.

Strategy Compare the expression for the voltage output just given with the general form, $\Delta v = \Delta V_{max} \sin 2\pi ft$, finding the maximum voltage. Substitute this result into the expression for the rms voltage.

Solution

Obtain the maximum voltage by comparison of the given expression for the output with the general expression:

$$\Delta v = (2.00 \times 10^2\ \text{V})\ \sin 2\pi ft \qquad \Delta v = \Delta V_{\text{max}} \sin 2\pi ft$$

$$\rightarrow \quad \Delta V_{\text{max}} = 2.00 \times 10^2\ \text{V}$$

Next, substitute into Equation 21.3 to find the rms voltage of the source:

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}} = \frac{2.00 \times 10^2\ \text{V}}{\sqrt{2}} = 141\ \text{V}$$

Substitute this result into Ohm's law to find the rms current:

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{R} = \frac{141\ \text{V}}{1.00 \times 10^2\ \Omega} = 1.41\ \text{A}$$

Remark Notice how the concept of rms values allows the handling of an AC circuit quantitatively in much the same way as a DC circuit.

QUESTION 21.1

True or False: The rms current in an AC circuit oscillates sinusoidally with time.

EXERCISE 21.1

Find the maximum current in the circuit and the average power delivered to the circuit.

Answer 2.00 A; 2.00×10^2 W

APPLYING PHYSICS 21.1 ELECTRIC FIELDS AND CANCER TREATMENT

Cancer cells multiply far more frequently than most normal cells, spreading throughout the body, using its resources and interfering with normal functioning. Most therapies damage both cancerous and healthy cells, so finding methods that target cancer cells is important in developing better treatments for the disease.

Because cancer cells multiply so rapidly, it's natural to consider treatments that prevent or disrupt cell division. Treatments such as chemotherapy interfere with the cell division cycle, but can also damage healthy cells. It has recently been found that alternating electric fields produced by AC currents in the range of 100 kHz can disrupt the cell division cycle, either by slowing the division or by causing a dividing cell to disintegrate. Healthy cells that divide at only a very slow rate are less vulnerable than the rapidly-dividing cancer cells, so such therapy holds out promise for certain types of cancer.

The alternating electric fields are thought to affect the process of mitosis, which is the dividing of the cell nucleus into two sets of identical chromosomes. Near the end of the first phase of mitosis, called the prophase, the mitotic spindle forms, a structure of fine filaments that guides the two replicated sets of chromosomes into separate daughter cells. The mitotic spindle is made up of a polymerization of dimers of tubulin, a protein with a large electric dipole moment. The alternating electric field exerts forces on these dipoles, disrupting their proper functioning.

Electric field therapy is especially promising for the treatment of brain tumors because healthy brain cells don't divide and therefore would be unharmed by the alternating electric fields. Research on such therapies is ongoing.

21.2 CAPACITORS IN AN AC CIRCUIT

To understand the effect of a capacitor on the behavior of a circuit containing an AC voltage source, we first review what happens when a capacitor is placed in a circuit containing a DC source, such as a battery. When the switch is closed in a series circuit containing a battery, a resistor, and a capacitor, the initial charge

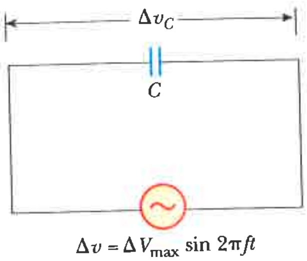


FIGURE 21.4 A series circuit consisting of a capacitor C connected to an AC generator.

on the plates of the capacitor is zero. The motion of charge through the circuit is therefore relatively free, and there is a large current in the circuit. As more charge accumulates on the capacitor, the voltage across it increases, opposing the current. After some time interval, which depends on the time constant RC, the current approaches zero. Consequently, a capacitor in a DC circuit limits or impedes the current so that it approaches zero after a brief time.

Now consider the simple series circuit in Figure 21.4, consisting of a capacitor connected to an AC generator. We sketch curves of current versus time and voltage versus time, and then attempt to make the graphs seem reasonable. The curves are shown in Figure 21.5. First, notice that the segment of the current curve from *a* to *b* indicates that the current starts out at a rather large value. This large value can be understood by recognizing that there is no charge on the capacitor at $t = 0$; as a consequence, there is nothing in the circuit except the resistance of the wires to hinder the flow of charge at this instant. The current decreases, however, as the voltage across the capacitor increases from *c* to *d* on the voltage curve. When the voltage is at point *d*, the current reverses and begins to increase in the opposite direction (from *b* to *e* on the current curve). During this time, the voltage across the capacitor decreases from *d* to *f* because the plates are now losing the charge they accumulated earlier. The remainder of the cycle for both voltage and current is a repeat of what happened during the first half of the cycle. The current reaches a maximum value in the opposite direction at point *e* on the current curve and then decreases as the voltage across the capacitor builds up.

In a purely resistive circuit, the current and voltage are always in step with each other. That isn't the case when a capacitor is in the circuit. In Figure 21.5, when an alternating voltage is applied across a capacitor, the voltage reaches its maximum value one-quarter of a cycle after the current reaches its maximum value. We say that **the voltage across a capacitor always lags the current by 90°**.

The voltage across a capacitor lags the current by 90° →

The impeding effect of a capacitor on the current in an AC circuit is expressed in terms of a factor called the **capacitive reactance** X_C, defined as

Capacitive reactance →

$$X_C \equiv \frac{1}{2\pi fC} \qquad [21.5]$$

When C is in farads and f is in hertz, the unit of X_C is the ohm. Notice that $2\pi f = \omega$, the angular frequency.

From Equation 21.5, as the frequency f of the voltage source increases, the capacitive reactance X_C (the impeding effect of the capacitor) decreases, so the current increases. At high frequency, there is less time available to charge the capacitor, so less charge and voltage accumulate on the capacitor, which translates into less opposition to the flow of charge and, consequently, a higher current. The analogy between capacitive reactance and resistance means that we can write an equation of the same form as Ohm's law to describe AC circuits containing capacitors. This equation relates the rms voltage and rms current in the circuit to the capacitive reactance:

$$\Delta V_{C,\text{rms}} = I_{\text{rms}} X_C \qquad [21.6]$$

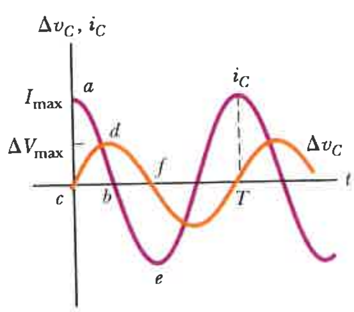


FIGURE 21.5 Plots of current and voltage across a capacitor versus time in an AC circuit. The voltage lags the current by 90°.

EXAMPLE 21.2 A Purely Capacitive AC Circuit

Goal Perform basic AC circuit calculations for a capacitive circuit.

Problem An 8.00-μF capacitor is connected to the terminals of an AC generator with an rms voltage of 1.50×10^2 V and a frequency of 60.0 Hz. Find the capacitive reactance and the rms current in the circuit.

Strategy Substitute values into Equations 21.5 and 21.6.

Solution

Substitute the values of f and C into Equation 21.5:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60.0\text{ Hz})(8.00 \times 10^{-6}\text{ F})} = 332\ \Omega$$

Solve Equation 21.6 for the current and substitute the values for X_C and the rms voltage to find the rms current:

$$I_{\text{rms}} = \frac{\Delta V_{C,\text{rms}}}{X_C} = \frac{1.50 \times 10^2\text{ V}}{332\ \Omega} = 0.452\text{ A}$$

Remark Again, notice how similar the technique is to that of analyzing a DC circuit with a resistor.

QUESTION 21.2

True or False: The larger the capacitance of a capacitor, the larger the capacitive reactance.

EXERCISE 21.2

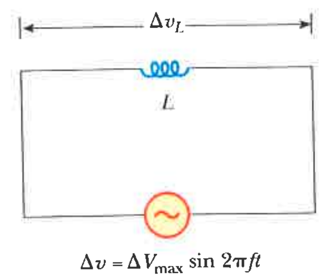
If the frequency is doubled, what happens to the capacitive reactance and the rms current?

Answer X_C is halved, and I_{rms} is doubled.

21.3 INDUCTORS IN AN AC CIRCUIT

Now consider an AC circuit consisting only of an inductor connected to the terminals of an AC source, as in Active Figure 21.6. (In any real circuit there is some resistance in the wire forming the inductive coil, but we ignore this consideration for now.) The changing current output of the generator produces a back emf that impedes the current in the circuit. The magnitude of this back emf is

$$\Delta v_L = L\frac{\Delta I}{\Delta t} \qquad [21.7]$$



ACTIVE FIGURE 21.6
A series circuit consisting of an inductor L connected to an AC generator.

The effective resistance of the coil in an AC circuit is measured by a quantity called the **inductive reactance**, X_L:

$$X_L \equiv 2\pi fL \qquad [21.8]$$

When f is in hertz and L is in henries, the unit of X_L is the ohm. The inductive reactance *increases* with increasing frequency and increasing inductance. Contrast these facts with capacitors, where increasing frequency or capacitance *decreases* the capacitive reactance.

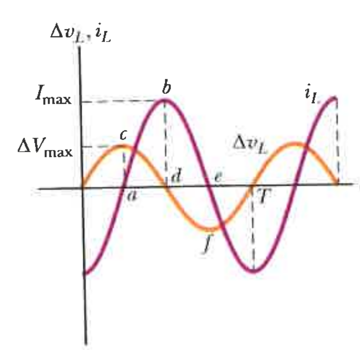
To understand the meaning of inductive reactance, compare Equation 21.8 with Equation 21.7. First, note from Equation 21.8 that the inductive reactance depends on the inductance L, which is reasonable because the back emf (Eq. 21.7) is large for large values of L. Second, note that the inductive reactance depends on the frequency f. This dependence, too, is reasonable because the back emf depends on $\Delta I/\Delta t$, a quantity that is large when the current changes rapidly, as it would for high frequencies.

With inductive reactance defined in this way, we can write an equation of the same form as Ohm's law for the voltage across the coil or inductor:

$$\Delta V_{L,\text{rms}} = I_{\text{rms}} X_L \qquad [21.9]$$

where $\Delta V_{L,\text{rms}}$ is the rms voltage across the coil and I_{rms} is the rms current in the coil.

Active Figure 21.7 shows the instantaneous voltage and instantaneous current across the coil as functions of time. When a sinusoidal voltage is applied across an inductor, the voltage reaches its maximum value one-quarter of an oscillation period before the current reaches its maximum value. In this situation we say that **the voltage across an inductor always leads the current by 90°**.



ACTIVE FIGURE 21.7
Plots of current and voltage across an inductor versus time in an AC circuit. The voltage leads the current by 90°.

To see why there is a phase relationship between voltage and current, we examine a few points on the curves of Active Figure 21.7. At point a on the current curve, the current is beginning to increase in the positive direction. At this instant the rate of change of current, $\Delta I/\Delta t$ (the slope of the current curve), is at a maximum, and we see from Equation 21.7 that the voltage across the inductor is consequently also at a maximum. As the current rises between points a and b on the curve, $\Delta I/\Delta t$ gradually decreases until it reaches zero at point b. As a result, the voltage across the inductor is decreasing during this same time interval, as the segment between c and d on the voltage curve indicates. Immediately after point b, the current begins to decrease, although it still has the same direction it had during the previous quarter cycle. As the current decreases to zero (from b to e on the curve), a voltage is again induced in the coil (from d to f), but the polarity of this voltage is opposite the polarity of the voltage induced between c and d. This occurs because back emfs always oppose the change in the current.

We could continue to examine other segments of the curves, but no new information would be gained because the current and voltage variations are repetitive.

EXAMPLE 21.3 A Purely Inductive AC Circuit

Goal Perform basic AC circuit calculations for an inductive circuit.

Problem In a purely inductive AC circuit (see Active Fig. 21.6), $L = 25.0$ mH and the rms voltage is 1.50×10^2 V. Find the inductive reactance and rms current in the circuit if the frequency is 60.0 Hz.

Solution

Substitute L and f into Equation 21.8 to get the inductive reactance:

$$X_L = 2\pi f L = 2\pi(60.0\ \text{s}^{-1})(25.0 \times 10^{-3}\ \text{H}) = 9.42\ \Omega$$

Solve Equation 21.9 for the rms current and substitute:

$$I_{\text{rms}} = \frac{\Delta V_{L,\text{rms}}}{X_L} = \frac{1.50 \times 10^2\ \text{V}}{9.42\ \Omega} = 15.9\ \text{A}$$

Remark The analogy with DC circuits is even closer than in the capacitive case because in the inductive equivalent of Ohm's law, the voltage across an inductor is *proportional* to the inductance L, just as the voltage across a resistor is proportional to R in Ohm's law.

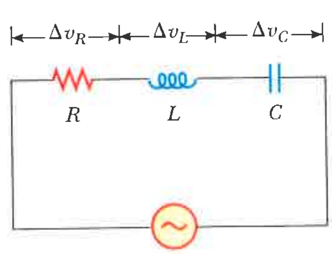
QUESTION 21.3

True or False: A larger inductance or frequency results in a larger inductive reactance.

EXERCISE 21.3

Calculate the inductive reactance and rms current in a similar circuit if the frequency is again 60.0 Hz, but the rms voltage is 85.0 V and the inductance is 47.0 mH.

Answer $X_L = 17.7\ \Omega$, $I = 4.80$ A



ACTIVE FIGURE 21.8
A series circuit consisting of a resistor, an inductor, and a capacitor connected to an AC generator.

21.4 THE *RLC* SERIES CIRCUIT

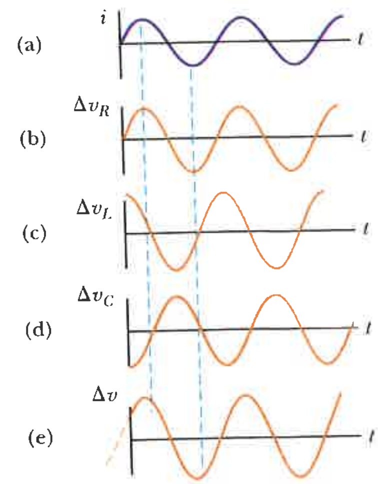
In the foregoing sections we examined the effects of an inductor, a capacitor, and a resistor when they are connected separately across an AC voltage source. We now consider what happens when these elements are combined.

Active Figure 21.8 shows a circuit containing a resistor, an inductor, and a capacitor connected in series across an AC source that supplies a total voltage Δv at some instant. The current in the circuit is the same at all points in the circuit at any instant and varies sinusoidally with time, as indicated in Active Figure 21.9a. This fact can be expressed mathematically as

$$i = I_{\max} \sin 2\pi f t$$

Earlier, we learned that the voltage across each element may or may not be in phase with the current. The instantaneous voltages across the three elements, shown in Active Figure 21.9, have the following phase relations to the instantaneous current:

1. The instantaneous voltage Δv_R across the resistor is *in phase* with the instantaneous current. (See Active Fig. 21.9b.)
2. The instantaneous voltage Δv_L across the inductor *leads* the current by 90°. (See Active Fig. 21.9c.)
3. The instantaneous voltage Δv_C across the capacitor *lags* the current by 90°. (See Active Fig. 21.9d.)



ACTIVE FIGURE 21.9
Phase relations in the series *RLC* circuit shown in Active Figure 21.8.

The net instantaneous voltage Δv supplied by the AC source equals the sum of the instantaneous voltages across the separate elements: $\Delta v = \Delta v_R + \Delta v_C + \Delta v_L$. This doesn't mean, however, that the voltages measured with an AC voltmeter across R, C, and L sum to the measured source voltage! In fact, the measured voltages *don't* sum to the measured source voltage because the voltages across R, C, and L all have different phases.

To account for the different phases of the voltage drops, we use a technique involving vectors. We represent the voltage across each element with a rotating vector, as in Figure 21.10. The rotating vectors are referred to as **phasors**, and the diagram is called a **phasor diagram**. This particular diagram represents the circuit voltage given by the expression $\Delta v = \Delta V_{\max} \sin(2\pi f t + \phi)$, where $\Delta V_{\max}$ is the maximum voltage (the magnitude or length of the rotating vector or phasor) and ϕ is the angle between the phasor and the positive x-axis when $t = 0$. The phasor can be viewed as a vector of magnitude $\Delta V_{\max}$ rotating at a constant frequency f so that its projection along the y-axis is the instantaneous voltage in the circuit. Because ϕ is the phase angle between the voltage and current in the circuit, the phasor for the current (not shown in Fig. 21.10) lies along the positive x-axis when $t = 0$ and is expressed by the relation $i = I_{\max} \sin(2\pi f t)$.

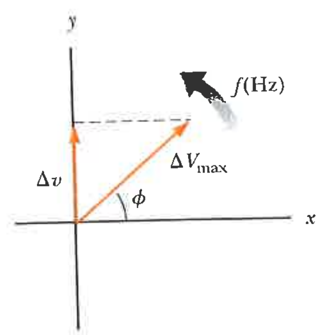


FIGURE 21.10 A phasor diagram for the voltage in an AC circuit, where ϕ is the phase angle between the voltage and the current and Δv is the instantaneous voltage.

The phasor diagrams in Figure 21.11 are useful for analyzing the *series RLC* circuit. Voltages in phase with the current are represented by vectors along the positive x-axis, and voltages out of phase with the current lie along other directions. ΔV_R is horizontal and to the right because it's in phase with the current. Likewise, ΔV_L is represented by a phasor along the positive y-axis because it leads the current by 90°. Finally, ΔV_C is along the negative y-axis because it lags the current[2] by 90°. If the phasors are added as vector quantities so as to account for the different phases of the voltages across R, L, and C, Figure 21.11a shows that the only x-component for the voltages is ΔV_R and the net y-component is $\Delta V_L - \Delta V_C$. We now add the phasors vectorially to find the phasor $\Delta V_{\max}$ (Fig. 21.11b), which represents the maximum voltage.

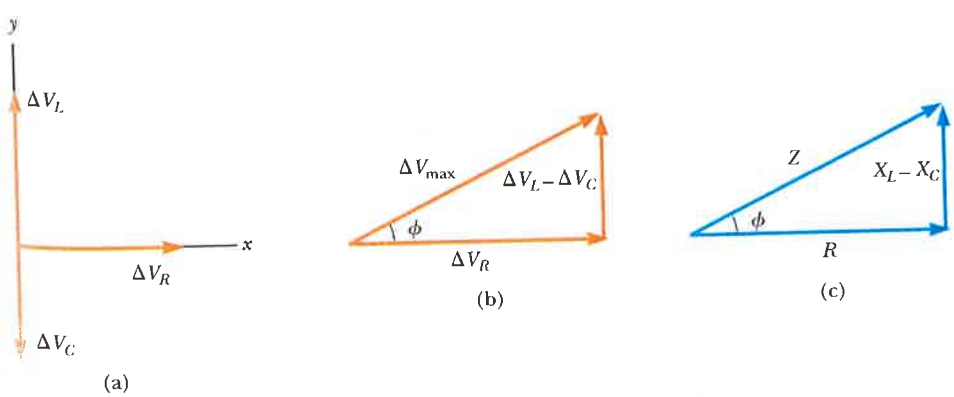


FIGURE 21.11 (a) A phasor diagram for the *RLC* circuit. (b) Addition of the phasors as vectors gives $\Delta V_{\max} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2}$. (c) The reactance triangle that gives the impedance relation $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

[2]A mnemonic to help you remember the phase relationships in *RLC* circuits is "*ELI* the *ICE* man." *E* represents the voltage $\mathcal{E}$, *I* the current, *L* the inductance, and *C* the capacitance. Thus, the name *ELI* means that in an inductive circuit, the voltage $\mathcal{E}$ leads the current *I*. In a capacitive circuit *ICE* means that the current leads the voltage.

The right triangle in Figure 21.11b gives the following equations for the maximum voltage and the phase angle ϕ between the maximum voltage and the current:

$$\Delta V_{\max} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2} \quad [21.10]$$

$$\tan \phi = \frac{\Delta V_L - \Delta V_C}{\Delta V_R} \quad [21.11]$$

In these equations, all voltages are maximum values. Although we choose to use maximum voltages in our analysis, the preceding equations apply equally well to rms voltages because the two quantities are related to each other by the same factor for all circuit elements. The result for the maximum voltage $\Delta V_{\max}$ given by Equation 21.10 reinforces the fact that **the voltages across the resistor, capacitor, and inductor are not in phase, so one cannot simply add them to get the voltage across the combination of element or to get the source voltage.**

QUICK QUIZ 21.2 For the circuit of Figure 21.8, is the instantaneous voltage of the source equal to (a) the sum of the maximum voltages across the elements, (b) the sum of the instantaneous voltages across the elements, or (c) the sum of the rms voltages across the elements?

We can write Equation 21.10 in the form of Ohm's law, using the relations $\Delta V_R = I_{\max} R$, $\Delta V_L = I_{\max} X_L$, and $\Delta V_C = I_{\max} X_C$, where $I_{\max}$ is the maximum current in the circuit:

$$\Delta V_{\max} = I_{\max}\sqrt{R^2 + (X_L - X_C)^2} \quad [21.12]$$

It's convenient to define a parameter called the **impedance** Z of the circuit as

Impedance →

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2} \quad [21.13]$$

so that Equation 21.12 becomes

$$\Delta V_{\max} = I_{\max} Z \quad [21.14]$$

Equation 21.14 is in the form of Ohm's law, $\Delta V = IR$, with R replaced by the impedance in ohms. Indeed, Equation 21.14 can be regarded as a generalized form of Ohm's law applied to a series AC circuit. Both the impedance and therefore the current in an AC circuit depend on the resistance, the inductance, the capacitance, *and* the frequency (because the reactances are frequency dependent).

It's useful to represent the impedance Z with a vector diagram such as the one depicted in Figure 21.11c. A right triangle is constructed with right side $X_L - X_C$, base R, and hypotenuse Z. Applying the Pythagorean theorem to this triangle, we see that

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

which is Equation 21.13. Furthermore, we see from the vector diagram in Figure 21.11c that the phase angle ϕ between the current and the voltage obeys the relationship

Phase angle ϕ →

$$\tan \phi = \frac{X_L - X_C}{R} \quad [21.15]$$

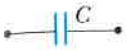
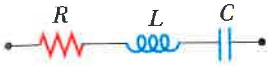
The physical significance of the phase angle will become apparent in Section 21.5.

Table 21.2 provides impedance values and phase angles for some series circuits containing different combinations of circuit elements.

Parallel alternating current circuits are also useful in everyday applications. We won't discuss them here, however, because their analysis is beyond the scope of this book.

TABLE 21.2

Impedance Values and Phase Angles for Various Combinations of Circuit Elements

Circuit Elements	Impedance Z	Phase Angle ϕ
 R	R	0°
 C	X_C	–90°
 L	X_L	+90°
 R C	$\sqrt{R^2 + X_C^2}$	Negative, between –90° and 0°
 R L	$\sqrt{R^2 + X_L^2}$	Positive, between 0° and 90°
 R L C	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

Note: In each case an AC voltage (not shown) is applied across the combination of elements (that is, across the dots).

QUICK QUIZ 21.3 If switch A is closed in Figure 21.12, what happens to the impedance of the circuit? (a) It increases. (b) It decreases. (c) It doesn't change.

QUICK QUIZ 21.4 Suppose $X_L > X_C$. If switch A is closed in Figure 21.12, what happens to the phase angle? (a) It increases. (b) It decreases. (c) It doesn't change.

QUICK QUIZ 21.5 Suppose $X_L > X_C$. If switch A is left open and switch B is closed in Figure 21.12, what happens to the phase angle? (a) It increases. (b) It decreases. (c) It doesn't change.

QUICK QUIZ 21.6 Suppose $X_L > X_C$ in Figure 21.12 and, with both switches open, a piece of iron is slipped into the inductor. During this process, what happens to the brightness of the bulb? (a) It increases. (b) It decreases. (c) It doesn't change.

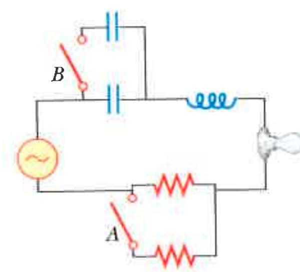


FIGURE 21.12 (Quick Quizzes 21.3–21.6)

PROBLEM-SOLVING STRATEGY

RLC CIRCUITS

The following procedure is recommended for solving series *RLC* circuit problems:

1. Calculate the inductive and capacitive reactances, X_L and X_C.
2. Use X_L and X_C together with the resistance R to calculate the impedance Z of the circuit.
3. Find the maximum current or maximum voltage drop with the equivalent of Ohm's law, $\Delta V_{\max} = I_{\max} Z$.
4. Calculate the voltage drops across the individual elements with the appropriate variations of Ohm's law: $\Delta V_{R,\max} = I_{\max} R$, $\Delta V_{L,\max} = I_{\max} X_L$, and $\Delta V_{C,\max} = I_{\max} X_C$.
5. Obtain the phase angle using $\tan \phi = (X_L - X_C)/R$.

Bettmann/Corbis

NIKOLA TESLA
(1856–1943)

Tesla was born in Croatia, but spent most of his professional life as an inventor in the United States. He was a key figure in the development of alternating-current electricity, high-voltage transformers, and the transport of electrical power via AC transmission lines. Tesla's viewpoint was at odds with the ideas of Edison, who committed himself to the use of direct current in power transmission. Tesla's AC approach won out.

EXAMPLE 21.4 An *RLC* Circuit

Goal Analyze a series *RLC* AC circuit and find the phase angle.

Problem A series *RLC* AC circuit has resistance $R = 2.50 \times 10^2\ \Omega$, inductance $L = 0.600$ H, capacitance $C = 3.50\ \mu$F, frequency $f = 60.0$ Hz, and maximum voltage $\Delta V_{\max} = 1.50 \times 10^2$ V. Find **(a)** the impedance of the circuit, **(b)** the maximum current in the circuit, **(c)** the phase angle, and **(d)** the maximum voltages across the elements.

Strategy Calculate the inductive and capacitive reactances, which can be used with the resistance to calculate the impedance and phase angle. The impedance and Ohm's law yield the maximum current.

Solution

(a) Find the impedance of the circuit.

First, calculate the inductive and capacitive reactances:

$$X_L = 2\pi f L = 226\ \Omega \qquad X_C = 1/2\pi f C = 758\ \Omega$$

Substitute these results and the resistance *R* into Equation 21.13 to obtain the impedance of the circuit:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(2.50 \times 10^2\ \Omega)^2 + (226\ \Omega - 758\ \Omega)^2} = 588\ \Omega$$

(b) Find the maximum current in the circuit.

Use Equation 21.12, the equivalent of Ohm's law, to find the maximum current:

$$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{1.50 \times 10^2\ \text{V}}{588\ \Omega} = 0.255\ \text{A}$$

(c) Find the phase angle.

Calculate the phase angle between the current and the voltage with Equation 21.15:

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\left(\frac{226\ \Omega - 758\ \Omega}{2.50 \times 10^2\ \Omega}\right) = -64.8^\circ$$

(d) Find the maximum voltages across the elements.

Substitute into the "Ohm's law" expressions for each individual type of current element:

$$\Delta V_{R,\max} = I_{\max} R = (0.255\ \text{A})(2.50 \times 10^2\ \Omega) = 63.8\ \text{V}$$

$$\Delta V_{L,\max} = I_{\max} X_L = (0.255\ \text{A})(2.26 \times 10^2\ \Omega) = 57.6\ \text{V}$$

$$\Delta V_{C,\max} = I_{\max} X_C = (0.255\ \text{A})(7.58 \times 10^2\ \Omega) = 193\ \text{V}$$

Remarks Because the circuit is more capacitive than inductive ($X_C > X_L$), ϕ is negative. A negative phase angle means that the current leads the applied voltage. Notice also that the sum of the maximum voltages across the elements is $\Delta V_R + \Delta V_L + \Delta V_C = 314$ V, which is much greater than the maximum voltage of the generator, 150 V. As we saw in Quick Quiz 21.2, the sum of the maximum voltages is a meaningless quantity because when alternating voltages are added, *both their amplitudes and their phases* must be taken into account. We know that the maximum voltages across the various elements occur at different times, so it doesn't make sense to add all the maximum values. The correct way to "add" the voltages is through Equation 21.10.

QUESTION 21.4

True or False: In an *RLC* circuit, the impedance must always be greater than or equal to the resistance.

EXERCISE 21.4

Analyze a series *RLC* AC circuit for which $R = 175\ \Omega$, $L = 0.500$ H, $C = 22.5\ \mu$F, $f = 60.0$ Hz, and $\Delta V_{\max} = 325$ V. Find (a) the impedance, (b) the maximum current, (c) the phase angle, and (d) the maximum voltages across the elements.

Answers (a) 189 Ω (b) 1.72 A (c) 22.0° (d) $\Delta V_{R,\max} = 301$ V, $\Delta V_{L,\max} = 324$ V, $\Delta V_{C,\max} = 203$ V

21.5 POWER IN AN AC CIRCUIT

No power losses are associated with pure capacitors and pure inductors in an AC circuit. A pure capacitor, by definition, has no resistance or inductance, whereas a pure inductor has no resistance or capacitance. (These definitions are idealizations: in a real capacitor, for example, inductive effects could become important at high frequencies.) We begin by analyzing the power dissipated in an AC circuit that contains only a generator and a capacitor.

When the current increases in one direction in an AC circuit, charge accumulates on the capacitor and a voltage drop appears across it. When the voltage reaches its maximum value, the energy stored in the capacitor is

$$PE_C = \tfrac{1}{2}C(\Delta V_{\text{max}})^2$$

This energy storage is only momentary, however: When the current reverses direction, the charge leaves the capacitor plates and returns to the voltage source. During one-half of each cycle the capacitor is being charged, and during the other half the charge is being returned to the voltage source. Therefore, the average power supplied by the source is zero. In other words, **no power losses occur in a capacitor in an AC circuit.**

Similarly, the source must do work against the back emf of an inductor that is carrying a current. When the current reaches its maximum value, the energy stored in the inductor is a maximum and is given by

$$PE_L = \tfrac{1}{2}LI_{\text{max}}^2$$

When the current begins to decrease in the circuit, this stored energy is returned to the source as the inductor attempts to maintain the current in the circuit. The average power delivered to a resistor in an *RLC* circuit is

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}^2 R \qquad \textbf{[21.16]}$$

The average power delivered by the generator is converted to internal energy in the resistor. No power loss occurs in an ideal capacitor or inductor.

An alternate equation for the average power loss in an AC circuit can be found by substituting (from Ohm's law) $R = \Delta V_{R,\text{rms}}/I_{\text{rms}}$ into Equation 21.16:

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}\,\Delta V_{R,\text{rms}}$$

It's convenient to refer to a voltage triangle that shows the relationship among ΔV_{rms}, $\Delta V_{R,\text{rms}}$, and $\Delta V_{L,\text{rms}} - \Delta V_{C,\text{rms}}$, such as Figure 21.11b. (Remember that Fig. 21.11 applies to *both* maximum and rms voltages.) From this figure, we see that the voltage drop across a resistor can be written in terms of the voltage of the source, ΔV_{rms}:

$$\Delta V_{R,\text{rms}} = \Delta V_{\text{rms}} \cos\phi$$

Hence, the average power delivered by a generator in an AC circuit is

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}\,\Delta V_{\text{rms}} \cos\phi \qquad \textbf{[21.17]}$$

← Average power

where the quantity $\cos\phi$ is called the **power factor**.

APPLICATION

Shifting phase to deliver more power

Equation 21.17 shows that the power delivered by an AC source to any circuit depends on the phase difference between the source voltage and the resulting current. This fact has many interesting applications. For example, factories often use devices such as large motors in machines, generators, and transformers that have a large inductive load due to all the windings. To deliver greater power to such devices without using excessively high voltages, factory technicians introduce capacitance in the circuits to shift the phase.

EXAMPLE 21.5 Average Power in an *RLC* Series Circuit

Goal Understand power in *RLC* series circuits.

Problem Calculate the average power delivered to the series *RLC* circuit described in Example 21.4.

Strategy After finding the rms current and rms voltage with Equations 21.2 and 21.3, substitute into Equation 21.17, using the phase angle found in Example 21.4.

Solution

First, use Equations 21.2 and 21.3 to calculate the rms current and rms voltage:

$$I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} = \frac{0.255\text{ A}}{\sqrt{2}} = 0.180\text{ A}$$

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}} = \frac{1.50 \times 10^2\text{ V}}{\sqrt{2}} = 106\text{ V}$$

Substitute these results and the phase angle $\phi = -64.8°$ into Equation 21.17 to find the average power:

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}\Delta V_{\text{rms}}\cos\phi = (0.180\text{ A})(106\text{ V})\cos(-64.8°)$$

$$= 8.12\text{ W}$$

Remark The same result can be obtained from Equation 21.16, $\mathcal{P}_{\text{av}} = I_{\text{rms}}^2 R$.

QUESTION 21.5

Under what circumstance can the average power of an *RLC* circuit be zero?

EXERCISE 21.5

Repeat this problem, using the system described in Exercise 21.4.

Answer 259 W

21.6 RESONANCE IN A SERIES *RLC* CIRCUIT

In general, the rms current in a series *RLC* circuit can be written

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}} \quad [21.18]$$

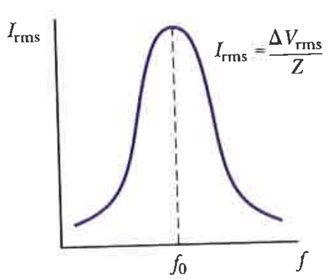


FIGURE 21.13 A plot of current amplitude in a series *RLC* circuit versus frequency of the generator voltage. Notice that the current reaches its maximum value at the resonance frequency f_0.

From this equation, we see that if the frequency is varied, the current has its *maximum* value when the impedance has its *minimum* value, which occurs when $X_L = X_C$. In such a circumstance, the impedance of the circuit reduces to $Z = R$. The frequency f_0 at which this happens is called the **resonance frequency** of the circuit. To find f_0, we set $X_L = X_C$, which gives, from Equations 21.5 and 21.8,

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

Resonance frequency →

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad [21.19]$$

Figure 21.13 is a plot of current as a function of frequency for a circuit containing a fixed value for both the capacitance and the inductance. From Equation 21.18, it must be concluded that the current would become infinite at resonance when $R = 0$. Although Equation 21.18 predicts this result, real circuits always have some resistance, which limits the value of the current.

APPLICATION

Tuning Your Radio

The tuning circuit of a radio is an important application of a series resonance circuit. The radio is tuned to a particular station (which transmits a specific radio-frequency signal) by varying a capacitor, which changes the resonance frequency of the tuning circuit. When this resonance frequency matches that of the incoming radio wave, the current in the tuning circuit increases.

APPLYING PHYSICS 21.2 METAL DETECTORS AT THE COURTHOUSE

When you walk through the doorway of a courthouse metal detector, as the person in Figure 21.14 is doing, you are really walking through a coil of many turns. How might the metal detector work?

Explanation The metal detector is essentially a resonant circuit. The portal you step through is an inductor (a large loop of conducting wire) that is part of the circuit. The frequency of the circuit is tuned to the resonant frequency of the circuit when there is no metal in the inductor. When you walk through with metal in your pocket, you change the effective inductance of the resonance circuit, resulting in a change in the current in the circuit. This change in current is detected, and an electronic circuit causes a sound to be emitted as an alarm.

Kira Vuille-Kowing

FIGURE 21.14 (Applying Physics 21.2) A courthouse metal detector.

EXAMPLE 21.6 A Circuit in Resonance

Goal Understand resonance frequency and its relation to inductance, capacitance, and the rms current.

Problem Consider a series *RLC* circuit for which $R = 1.50 \times 10^2\ \Omega$, $L = 20.0$ mH, $\Delta V_{rms} = 20.0$ V, and $f = 796\ \text{s}^{-1}$. **(a)** Determine the value of the capacitance for which the rms current is a maximum. **(b)** Find the maximum rms current in the circuit.

Strategy The current is a maximum at the resonance frequency f_0, which should be set equal to the driving frequency, 796 s^{-1}. The resulting equation can be solved for C. For part (b), substitute into Equation 21.18 to get the maximum rms current.

Solution

(a) Find the capacitance giving the maximum current in the circuit (the resonance condition).

Solve the resonance frequency for the capacitance:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \rightarrow \sqrt{LC} = \frac{1}{2\pi f_0} \rightarrow LC = \frac{1}{4\pi^2 f_0^{\,2}}$$

$$C = \frac{1}{4\pi^2 f_0^{\,2} L}$$

Insert the given values, substituting the source frequency for the resonance frequency, f_0:

$$C = \frac{1}{4\pi^2(796\ \text{Hz})^2(20.0 \times 10^{-3}\ \text{H})} = 2.00 \times 10^{-6}\ \text{F}$$

(b) Find the maximum rms current in the circuit.

The capacitive and inductive reactances are equal, so $Z = R = 1.50 \times 10^2\ \Omega$. Substitute into Equation 21.18 to find the rms current:

$$I_{rms} = \frac{\Delta V_{rms}}{Z} = \frac{20.0\ \text{V}}{1.50 \times 10^2\ \Omega} = 0.133\ \text{A}$$

Remark Because the impedance Z is in the denominator of Equation 21.18, the maximum current will always occur when $X_L = X_C$ because that yields the minimum value of Z.

QUESTION 21.6

True or False: The magnitude of the current in an *RLC* circuit is never larger than the rms current.