## Fluids

## I. Pascal's Principle

A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

## A. Submarine Example

The first term in our previous work demonstrates Pascal's principle. The pressure applied by air on the surface of the water, $\mathrm{P}_{0}$, is transferred to the submarine at any depth. The second term is the increase in pressure due to the weight of the water above the submarine pushing down upon the submarine and the water surrounding the submarine.


For the submarine, we see that the pressure is applied at all points around the submarine in order to ensure that the net external force is zero (i.e. it is not accelerating). We neglected this in our previous FBD work in order to simplify the drawing. For the submarine, the pressure would need to be greatest at the bottom since it must hold up the weight of the submarine and the submarine has a finite size. Our previous work suggests that this extra pressure must be the second term in our equation which only depends on the fluid!! This will lead us in the next section to Archimedes's principle which was known hundreds of years before Pascal's principle.

## B. Pressure on a Vessel

If instead of a submarine, we consider an infinitesimally small unit of volume then the pressure must be the same on all sides of the volume element. This fact has some interesting consequences. Consider an infinitesimal volume element of fluid next to the wall of a container as shown below.


The pressure applied on the left side of the volume element by the container must be the same as the pressure applied on the right side of the element by the rest of the fluid since the volume element can not accelerate. By Newton's third law, this also means that the volume element must apply the same pressure back up the container.

Pascal's principle combined with our knowledge that pressure in a fluid increases with depth implies that the pressure on the walls of a vessel also increases with depth.

Since the pressure on a dam increases at the bottom of the dam, the thickness of the dam must be larger at the bottom.


## C. Example - Hydraulic Lift

A useful application of Pascal's principle is the hydraulic lift. Consider an enclosed fluid shown below. A downward pressure of P is applied to a piston of area $\mathrm{A}_{1}$. This pressure is undiminished through the fluid (Pascal's principle) and pushes upward on a second piston on area $\mathrm{A}_{2}$.


Using the definition of pressure, we have

$$
\mathbf{P}=\quad=
$$

Thus, we see that the upward force on the second piston can be made large than the downward force $\mathrm{F}_{1}$ by the ratio of the areas of the two pistons.

$$
\mathbf{F}_{2}=
$$

This is the technique in which an auto mechanic can push down on a small piston and raise a car. It might seem sensible to think that the forces on the pistons would be the same. However, the force is increased because there are many more fluid atoms pushing on the larger area piston than on the smaller piston!!

