

## 1-D Motion Basic

### I. Definitions:

One dimensional motion (straight line) is a special case of motion where all but one vector component is zero. We will arrange our coordinate axis so that the x-axis lies along the direction of motion. Thus, we can simplify our general definitions to only involve the x-components as shown in the sections below. It is important to note that we are actually doing vector math in chapter 2. However, scalar math is a special case of vector math for one dimension so there is no difference between the types of math in this chapter!! This is not the case in future chapters where we will have to use trigonometry to break vectors into components.

#### A. Position - $x$

Using our definition of the position vector, we have  $\vec{r} = x\hat{i} + 0\hat{j} + 0\hat{k} = x\hat{i}$

The textbook chooses to say that the position vector is given by  $x$ . This is ok since the direction  $\hat{i}$  is understood. If  $x$  is negative then the particle is located in the  $-\hat{i}$  direction.

#### B. Displacement - $\Delta x$

Using our definition of the displacement vector, we have

$\Delta\vec{r} \equiv \vec{r}_2 - \vec{r}_1 = \Delta x\hat{i} + 0\hat{j} + 0\hat{k} = \Delta x\hat{i}$ . The textbook chooses to say that the displacement vector is given by  $\Delta x$ . This is ok since the direction  $\hat{i}$  is understood.

#### C. Average Velocity - $V_{av}$

Using our definition of the displacement vector, we have

$$\vec{V}_{av} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{i} + 0\hat{j} + 0\hat{k}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i}.$$

The textbook chooses to say that the average velocity is

$$V_{av} = \frac{\Delta x}{\Delta t}$$

This is ok since the direction  $\hat{i}$  is understood.

#### D. Instantaneous Velocity – $\vec{V}$

Using our definition of the displacement vector, we have

$$\vec{V} \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x \hat{i} + 0 \hat{j} + 0 \hat{k}}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} \hat{i}.$$

The textbook chooses to say that the velocity is

$$V = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

This is ok since the direction  $\hat{i}$  is understood.

#### E. Average Acceleration - $\vec{A}_{av}$

Using our definition of the displacement vector, we have

$$\vec{A}_{av} \equiv \frac{\Delta \vec{V}}{\Delta t} = \frac{\Delta V_x \hat{i} + 0 \hat{j} + 0 \hat{k}}{\Delta t} = \frac{\Delta V_x}{\Delta t} \hat{i}.$$

Since we only have x-components, the subscript on the change in velocity is not required. This gives us  $\vec{A}_{av} = \frac{\Delta V}{\Delta t} \hat{i}$ .

The textbook chooses to say that the average velocity is

$$A_{av} = \frac{\Delta V}{\Delta t}$$

This is ok since the direction  $\hat{i}$  is understood.

#### F. Instantaneous Acceleration - $\vec{A}$

Using our definition of the displacement vector, we have

$$\vec{A} \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{V}}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\Delta V_x \hat{i} + 0 \hat{j} + 0 \hat{k}}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\Delta V_x}{\Delta t} \hat{i}.$$

Since we only have x-components, the subscript on the change in velocity is not required. This gives us  $\vec{A} = \lim_{\Delta t \rightarrow 0} \frac{\Delta V}{\Delta t} \hat{i}$ .

The textbook chooses to say that the average velocity is

$$A = \lim_{\Delta t \rightarrow 0} \frac{\Delta V}{\Delta t}$$

This is ok since the direction  $\hat{i}$  is understood.

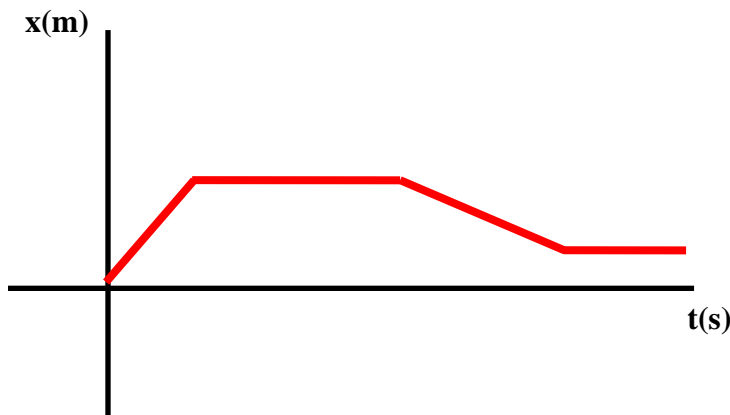
**NOTE: You don't have to learn any of these formulas so long as you know the vector definitions and can do vector math!!! You have to know the vector definitions**

to be successful in Chapter 3 and the rest of the course. Vector math skills will also help you through out the course. You only have a limited amount of time to study so spend your time wisely studying in a way that pays multiple dividends and not memorizing.

## II. Graphical Analysis of Motion

In order to analyze the motion of a particle using our definitions, we must collect data (information) about the particle that we wish to analyze. The most common data collected is the position of the particle as a function of time which we collect using our eyes. Other ways of collecting this information is to collect it using global positions satellites (GPS), radar, or a sonic range finder (motion sensor). For special cases like constant acceleration, we may have seen the graph so often that we are able to develop a formulas that allows us to solve for the slope of chords and tangent lines, and areas for the graph without graphing. However, these are just special results. Fundamentally, we can find everything from the motion graphs!!

### A. Position-Time Graph



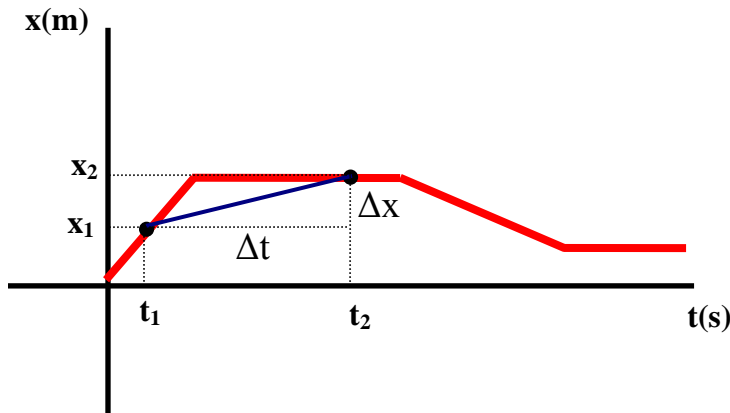
1. **Position** – To find the particles position at a particular time you just read the value from the graph using the y-axis.
2. **Displacement** – Read the particle's final and initial position using the graph and then use the definition of the displacement to find  $\Delta x$ .

**Example:** Find the ball's displacement over the time interval from  $t = 2s$  to  $t = 5s$ .

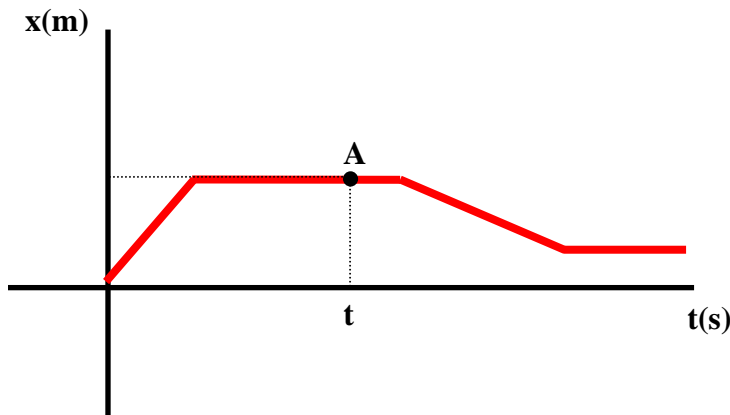
**Solution:** If we looked at the graph and found that the ball was located at  $x = 4m$  at  $t=2 s$  and  $x = 1 m$  and  $t = 5s$ , then the answer would be

$$\Delta x = x_2 - x_1 = 1m - 4m = -3m$$

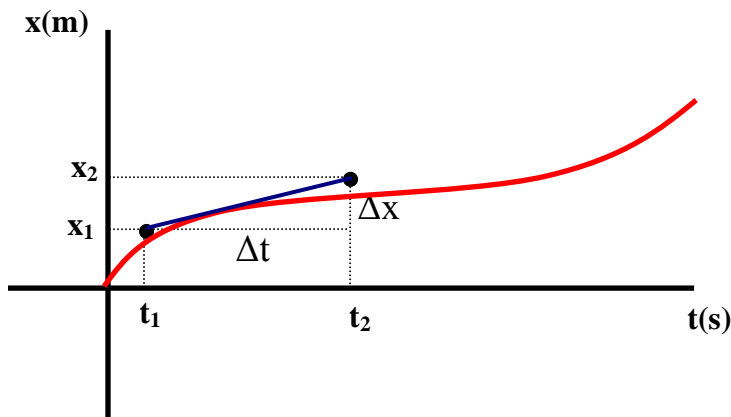
3. **Average Velocity** – The average velocity is found by using the definition of average velocity:  $V_{av} = \frac{\Delta x}{\Delta t}$ . Graphically, this is the formula for finding the slope of a chord connecting the initial location of the particle on the graph to the final location of the particle as shown below by the blue chord for the time interval  $t_1$  to  $t_2$ .



4. **Velocity** – The velocity at some time  $t$  is found by using the definition of velocity:  $V = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$ . Graphically, this is the formula for finding the slope of a tangent line. If the curve is a straight line at this instant in time then the curve is its own tangent line and the velocity is just the slope of the curve. For instance, the slope of the curve at point A and hence the velocity is zero as shown below.

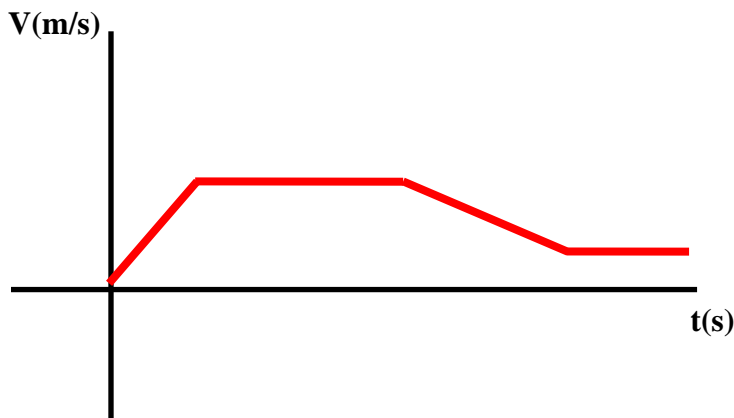


If the curve is not a straight line at the particular instant in question, then you must draw a tangent line as shown below in blue. The tangent line touches the curve at the instant in time that you want to find the velocity but doesn't cross the curve. You then find the slope of the tangent line that you have drawn.



5. **Acceleration** – In general the value for the acceleration can not be found directly from a position-time graph. You must first construct a velocity-time graph. The exception is if the acceleration is zero!! Anytime the position-time graph is a straight line then during that time interval, the velocity is constant and the acceleration is zero. If the position-time graph is not a straight line graph then the particle is accelerating!!

#### B. Velocity -Time Graph



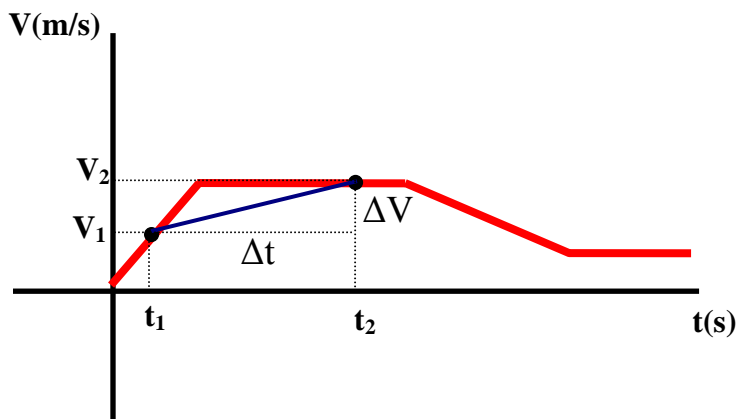
1. **Velocity** – To find the particle's velocity at a particular time you just read the value from the graph using the y-axis.
2. **Change in Velocity** – Read the particle's final and initial velocity using the graph and then use the definition of the change in velocity to find  $\Delta V$ .

**Example:** Find the ball's change in velocity over the time interval from  $t = 2\text{ s}$  to  $t = 5\text{ s}$ .

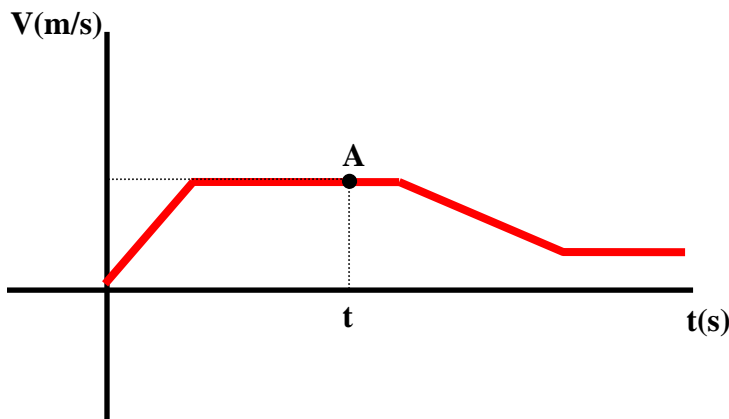
**Solution:** If we looked at the graph and found that the ball was moving at  $V = 4\text{ m/s}$  at  $t = 2\text{ s}$  and  $V = 1\text{ m/s}$  at  $t = 5\text{ s}$ , then the answer would be

$$\Delta V = V_2 - V_1 = 1\text{ m/s} - 4\text{ m/s} = -3\text{ m/s}$$

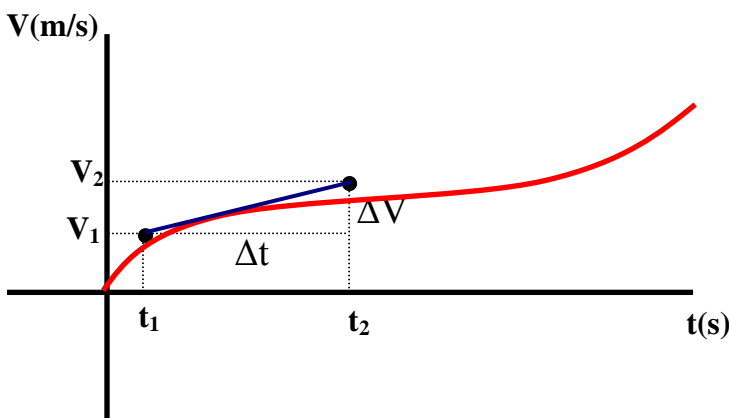
3. **Average Acceleration** – The average acceleration is found by using the definition of average acceleration:  $A_{\text{av}} = \frac{\Delta V}{\Delta t}$ . Graphically, this is the formula for finding the slope of a chord connecting the initial velocity of the particle on the graph to the final velocity of the particle as shown below by the blue chord for the time interval  $t_1$  to  $t_2$ .



4. **Acceleration** – The acceleration at some time  $t$  is found by using the definition of acceleration:  $A = \lim_{\Delta t \rightarrow 0} \frac{\Delta V}{\Delta t}$ . Graphically, this is the formula for finding the slope of a tangent line. If the curve is a straight line at this instant in time then the curve is its own tangent line and the acceleration is just the slope of the curve. For instance, the slope of the curve at point A and hence the acceleration is zero as shown below.

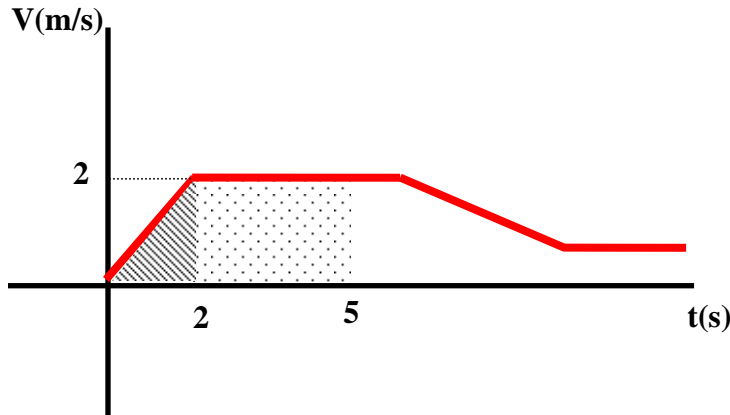


If the curve is not a straight line at the particular instant in question, then you must draw a tangent line as shown below in blue. The tangent line touches the curve at the instant in time that you want to find the acceleration but doesn't cross the curve. You then find the slope of the tangent line that you have drawn.



5. **Displacement** – The method for finding the displacement comes by solving the definition of velocity for displacement. The process where a limit is imposed upon a ratio as in the case of the definition of velocity is called differentiation and is one of the main topics of Calculus. Since this isn't a Calculus based course, it is enough for you to know that this is a process just like taking the square of a number. To undo a process, we must know the inverse process. For the squaring process, the inverse process is to take the square root. For differentiation, the inverse process is called integration and its value is the area between the curve and the horizontal axis. **Thus, displacement is the area between the curve of a velocity-time graph and the time axis. If the area is above the time axis then the area is positive. If the area is below the time axis, the area is negative.** Although Calculus gives tricks for finding areas under curves, you can always approximate the area by breaking the area up into a series of small triangles and rectangles. This is an application of a process called the “Method of Exhaustion” by the ancient Greeks.

**Example:** What was the particle's displacement over the time interval from  $t = 0\text{s}$  to  $t = 5\text{s}$ ?



**Solution:**

We break the area under the graph above into a triangle followed by a rectangle. The area under the curve and hence the displacement is found by

$$\Delta x = \frac{1}{2}(2\text{s})(2\text{ m/s}) + (3\text{s})(2\text{ m/s}) = 8\text{ m}$$

**6. Average Velocity** – To find the average velocity from a velocity-time graph, you must first find the displacement and then use the definition of average velocity.

**Example:** Find the particle's average velocity over the time interval from  $t = 0\text{ s}$  to  $t = 5\text{ s}$  for our last example.

**Solution:** Using our past results and the definition of average velocity, we have

$$V_{\text{av}} = \frac{\Delta x}{\Delta t} = \frac{8.0\text{ m}}{5.0\text{ s}} = 1.6 \frac{\text{m}}{\text{s}}$$

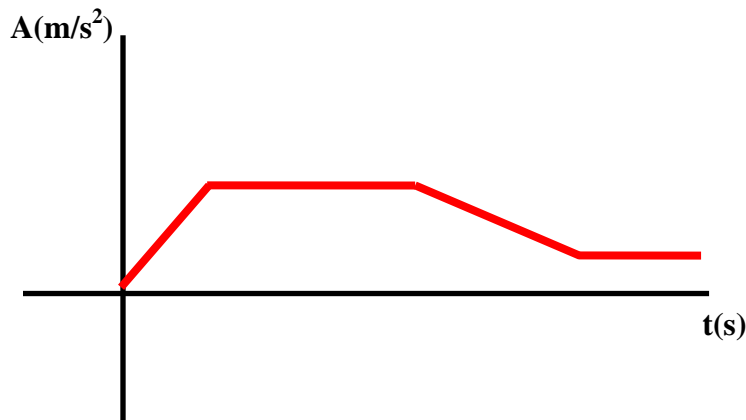
**NOTE:**  $V_{\text{av}} \neq \frac{V_2 + V_1}{2}$  except for the case where the velocity-time graph is a straight line (i.e. constant acceleration).

### C. Acceleration -Time Graph

We will see in Chapter 4 that the acceleration-time graph takes on special meaning in the field of physics because of the connection between acceleration and the external forces acting on the particle (Newton's 2<sup>nd</sup> Law). If we know the forces acting on a particle then we know the shape of the particle's acceleration-time curve even though we may not have measured its position or velocity. In this

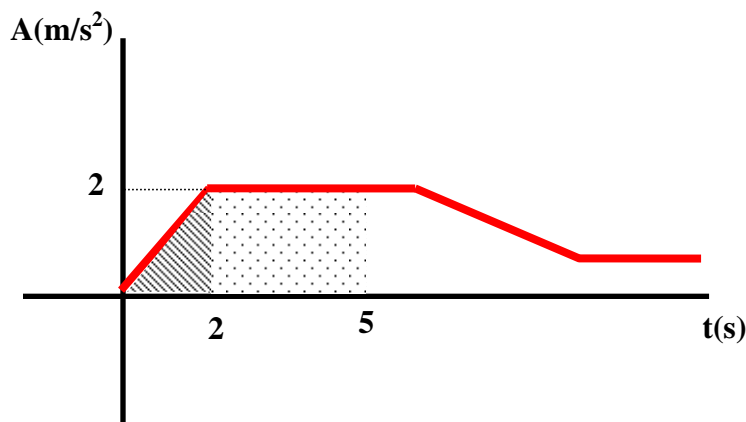


case we are working the problem backwards and trying to find out what the particle's velocity-time and position-time graph will look without measuring them. An example of this approach is the fact that we know the force of gravity acting on a planet so we know its acceleration-time graph. We can then use our definitions to determine the location and velocity of the planet in the future. Thus, the equations of physics in a sense allow us to act as a sort of fortune tellers to see the future as well as to be able to look into the past and locate the position of planets and stars.



1. **Acceleration** – To find the particle's acceleration at a particular time you just read the value from the graph using the y-axis.
2. **Change in Acceleration** – Read the particle's final and initial acceleration using the graph and then use the definition of the change in acceleration to find  $\Delta A$ .
5. **Change in Velocity** – The change in velocity is the area between the curve of a velocity-time graph and the time axis.

**Example:** What was the particle's change in velocity over the time interval from  $t = 0\text{s}$  to  $t = 5\text{s}$ ?



**Solution:**

We break the area under the graph above into a triangle followed by a rectangle. The area

under the curve and hence the displacement is found by

$$\Delta V = \frac{1}{2}(2\text{s})(2\text{ m/s}^2) + (3\text{s})(2\text{ m/s}^2) = 8\text{ m/s}$$

- 3. Average Acceleration** – To find the average acceleration from an acceleration-time graph, you must first find the displacement and then use the definition of average velocity.

**Example:** Find the particle's average acceleration over the time interval from  $t = 0\text{ s}$  to  $t = 5\text{ s}$  for our last example.

**Solution:** Using our past results and the definition of average acceleration, we have

$$A_{\text{av}} = \frac{\Delta V}{\Delta t} = \frac{8.0\text{ m/s}}{5.0\text{ s}} = 1.6\frac{\text{m}}{\text{s}^2}$$