

Gauss' Law

I. Introduction - Any vector field can be described completely if you know two properties of the field: 1) how it flows and 2) how it circulates.

Q. What makes Electric fields flow?

A. _____

Q. What makes static (non-time varying) electric fields circulate?

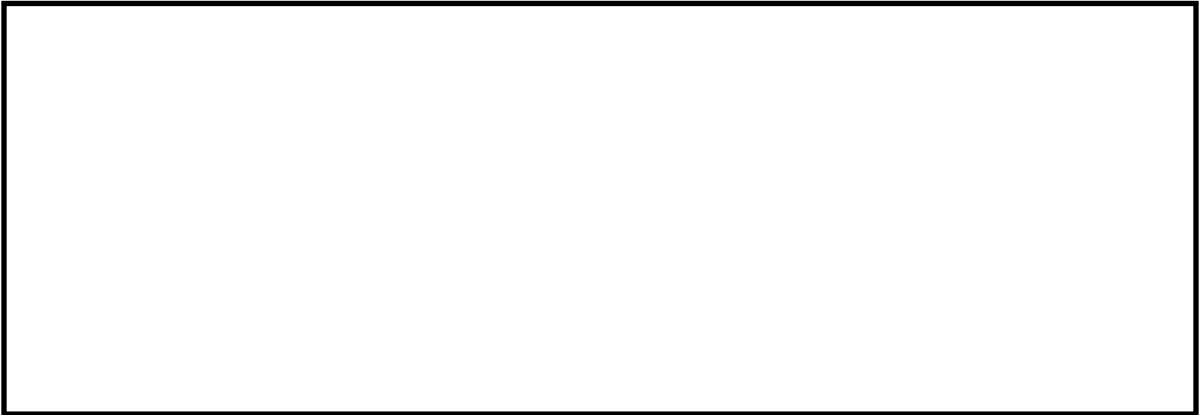
A. _____

II. Electric Flux - Φ_E

Review: Water Flux from the chapter on fluids

A. Definition of Electric Flux

The electric flux through any surface of area A is defined by the formula



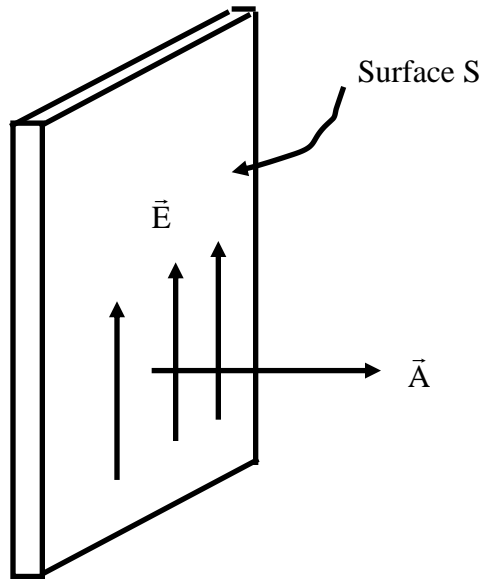
B. Physical Significance

Electric flux is a measure of the number of electric field lines penetrating some surface.

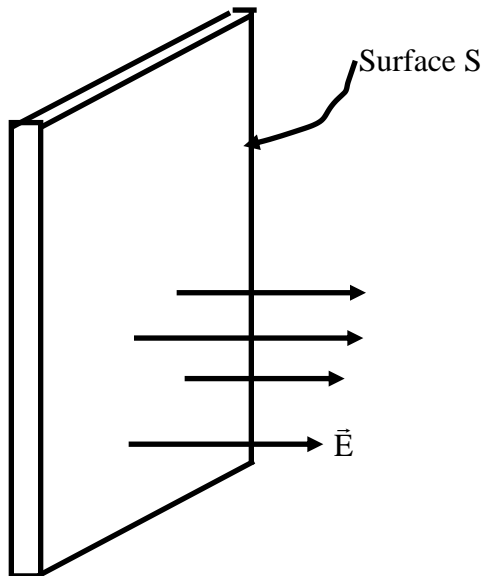
FACT:

The definition of electric flux always works, but since our goal is to find the Electric Field it isn't of much use unless the problem has a special symmetry as we will see with Gauss' Law.

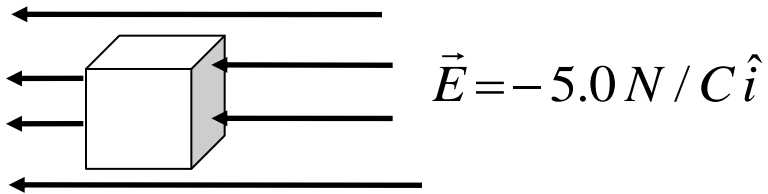
Inquiry I: What is the electric flux flowing through the right side of this surface? Make sure your answer agree both with the mathematical definition of electric flux and its physical interpretation.



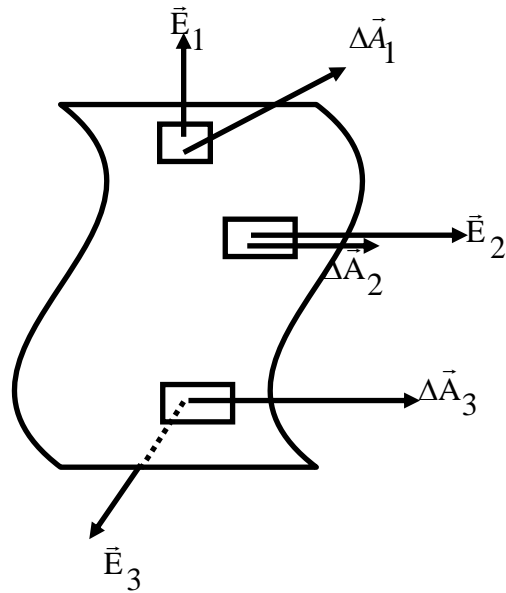
Inquiry II: What is the electric flux flowing through the right side of this surface? Make sure your answer agree both with the mathematical definition of electric flux and its physical interpretation.



Inquiry III: What is the total flux flowing through the closed surface of the cube shown below whose edges are 2.0 cm long? Make sure your answer agree both with the mathematical definition of electric flux and its physical interpretation.



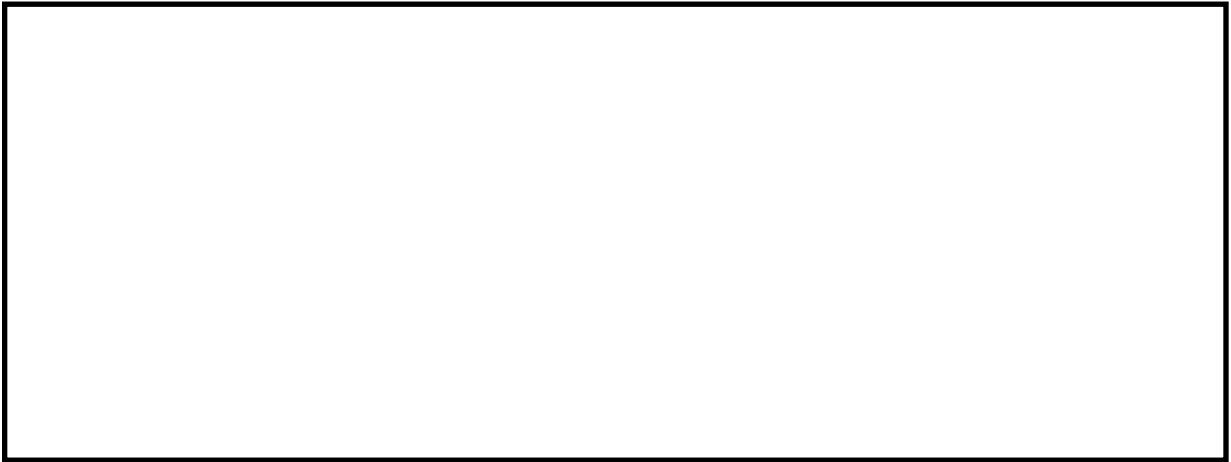
Tough Example: Irregular objects and non-uniform fields



We can add up the individual electric flux contribution in theory using a computer or approximate it by hand, but it would be very tough because the angle between the electric field and the area elements is different for each piece.

III. Gauss' Law for Electric Fields

- A.** The electric flux (proportional to the number of electric field lines) through a **CLOSED SURFACE** depends **ONLY** on the **NET CHARGE Enclosed** by the surface!!!



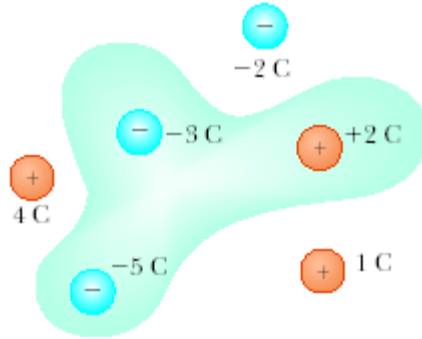
NOTE: Gauss' law is only valid for closed surfaces!!! It is more limited in scope than the definition of electric flux, but it is easier to apply since there is no summing.

Inquiry: What does Gauss' law say (if anything) about the net charge on each of the objects we had on the previous inquiries in the module?

B. Useful Facts

1. The electric flux through a closed surface depends only on the net charge enclosed by the surface and NOT on the shape of the surface!!!
2. Gauss' Law for Electric fields tells us the cause of "flow" in "electric fields." It is electric charge.
3. Gauss' Law can be useful in finding the magnitude of the electric field if you can find a surface where the $\cos \theta$ is either ± 1 or zero everywhere on the surface and where the magnitude of the electric field is constant. If you can't find such a surface then don't use Gauss' Law!!!
4. In Gauss' Law problems, the direction of the electric field is found by symmetry.

Inquiry: Find the electric flux through the blue surface shown below:



Example: A point charge of $+5.00\ \mu\text{C}$ is located at the center of a sphere with a radius of 12.0 cm . Determine the electric flux through the surface of the sphere.

IV. Using Gauss' Law to Calculate Electric Fields

1. Draw the pattern of the electric field lines for the charge distribution.
2. Choose an appropriate Gaussian surface. It should be chosen so that the electric field lines are either perpendicular with the same strength or are parallel to the surface at every point. It must also have the point where you want to calculate the electric field strength on the surface.

(Cylindrical objects → use cylindrical surface)

(Spherical objects → use spherical surface)

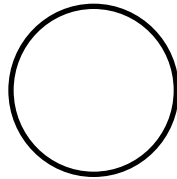
(Plate objects → use plane surface)

3. Find the area where the electric field is perpendicular to the surface.
4. Find the net charge enclosed by the Gaussian surface.
5. Solve Gauss' law to find the electric field strength.
6. Use symmetry to get the direction of the electric field.

NOTE: When a system has enough symmetry to make Gauss' Law practical for calculating the electric field, the final result for the electric field strength will reduce to:

$$E = \frac{1}{\epsilon_0} \left(\frac{Q_{\text{enclosed}}}{\text{Area of Gaussian Surface Through which Flux Flows}} \right)$$

Example: An solid spherical insulator of radius R is uniformly charged with a total charge of Q . Using Gauss' Law determine the electric field at a distance r from the center of the sphere where



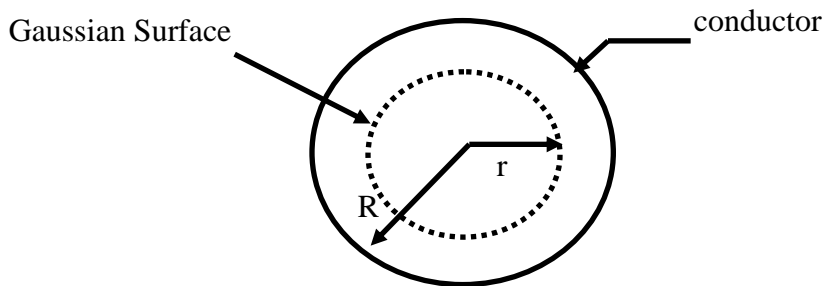
A) $r > R$ (i.e. Outside the sphere)

b) $r < R$ (inside the sphere)

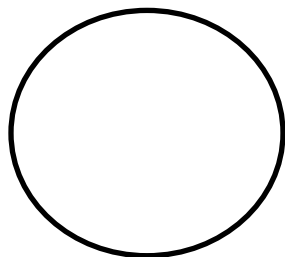
c) Graph the electric field as a function of radial distance r .

V. Conductors in Electrostatic Equilibrium

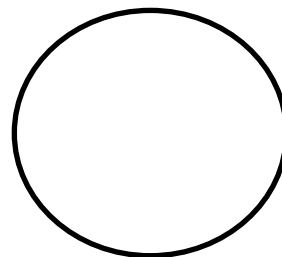
- A. The Electric Field is ZERO everywhere INSIDE a conductor.
- B. Any NET CHARGE must be on the SURFACE of the conductor.



- C. The electric field outside a conductor is PERPENDICULAR to the surface of the conductor and the magnitude of the electric field for a symmetric and uniformly charge conductor is $E = \sigma / \epsilon_0$.



"Not Static"



"Static"

REASON: A force due to the component of the electric field that is parallel to the conductor's surface would not be balanced by a force from the conductor!!

- D.** On irregular shaped conductors, the charge **accumulates** at **sharp points** (greater charge density). This is the principle behind devices designed to spark including spark gaps, and lightning rods. It is also important in preventing sparks from unwanted parts of a device like screws etc.

