# Center of Mass

**Definition:** The center of mass is a vector whose components are found by:

$$x_{cm} = \frac{\sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i} \qquad y_{cm} = \frac{\sum_{i=1}^{N} m_i y_i}{\sum_{i=1}^{N} m_i}$$

While the formula for the center of mass may look complicated, it is really just the mathematical way to write an average. Thus, finding the center-of-mass is similar to finding a class average on an exam. It is the average location of the mass that comprises a system. The position of each piece of mass is like a particular score on the test and the mass is similar to the number of people who made that particular score.

**Example:** The following scores were made on a 5 question exam. What is the class average?

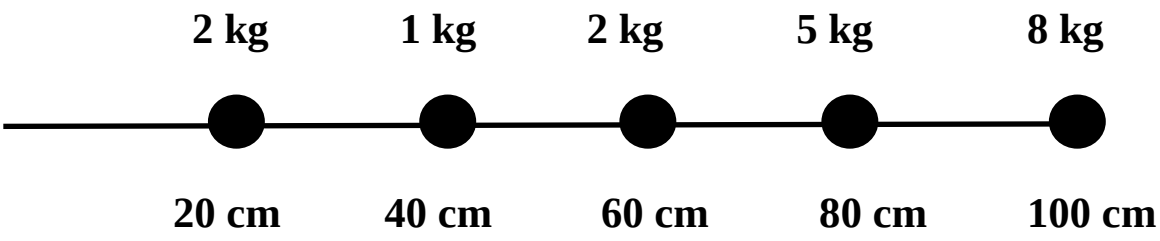
| Score | 20 | 40 | 60 | 80 | 100 |
|---|---|---|---|---|---|
| # of Students | 2 | 1 | 2 | 5 | 8 |

$$Average = \frac{2 * 20 + 1 * 40 + 2 * 60 + 5 * 80 + 8 * 100}{2 + 1 + 2 + 5 + 8}$$

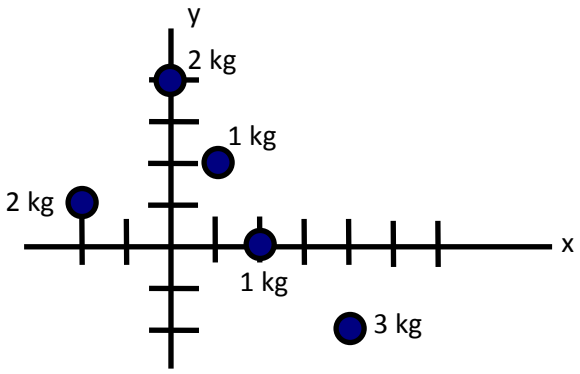
$$Average = \frac{1400}{18} = 77.8$$

Note: No student scored 77.8!! The average represents how the collection of students (class) did and not what a single individual did.

**Example 1:** Find the Center-of-Mass for the following system.




**Example 2:** Find the center of mass of the following system of particles.

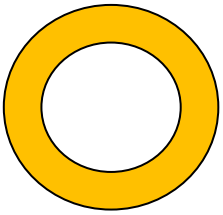


## Important Points

1. The center-of-mass of a system will often reside at a location in which no particle exists!!

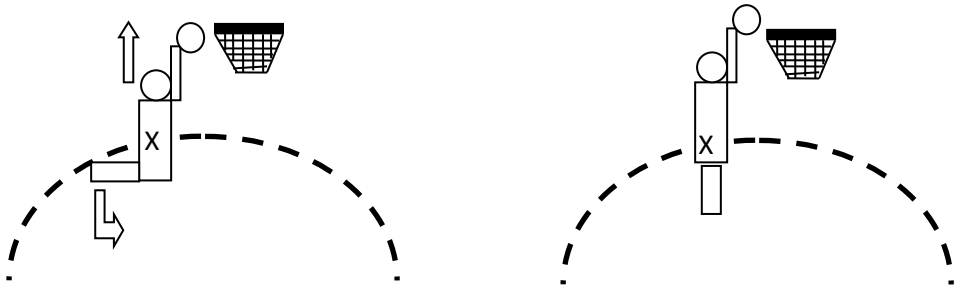
2. When an object is homogenous (uniform), you may be able to determine the center-of-mass without doing calculations by using symmetry.

**Example 3:** Find the center-of-mass of the toroid (donut).

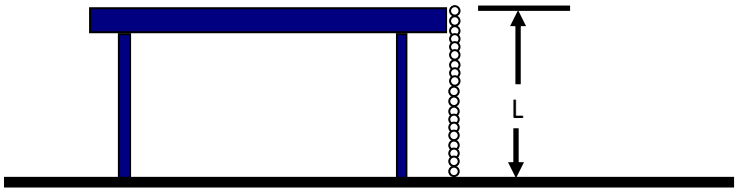


3. Analysis of compound objects or systems is often easier using the concept of the center of mass. The system acts as if it is a single particle with all its mass at the center-of-mass. Many events which may seem magical especially in sports are easily explained using the center of mass.

**Example 4:** Why does kicking out one's legs help you jump higher when trying to dunk a basketball?



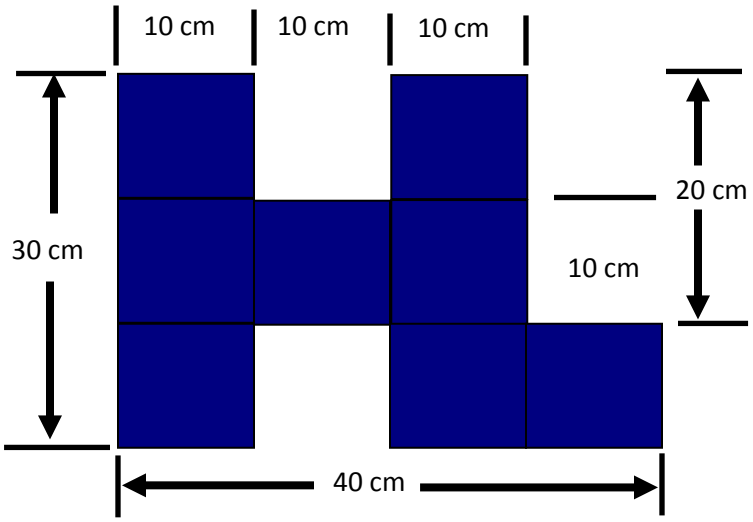
**Example 5:** A uniform chain of length L and mass M is hanging from a table. How much work would you have to do to raise the chain at constant speed onto the table?



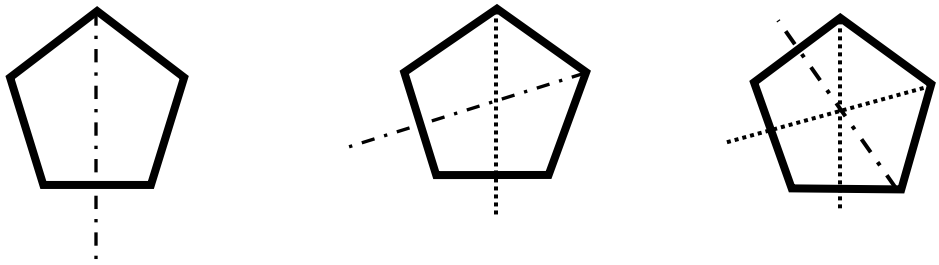

4. You can break apart complicated uniform objects into pieces. By replacing each piece with a particle with the same mass located at the piece's center-of-mass, one can calculate the center-of-mass for the whole object.

5. Holes can be treated as objects with negative mass.

**Example 6:** Find the center-of-mass of the following object.



**6.** You can also find the center of mass of irregular objects experimentally. For everyday objects on the Earth, the pull of gravity is approximately constant so the center of gravity (the effective point where the Earth pulls on an extended object) is the same point in space as the center of mass. By holding an object at different points and allowing it to rotate under the influence of gravity, it is possible to find the center of gravity.



This is also the point at which you could place a support to balance the object!! You may have seen acrobats who were balancing in strange formations or small toy birds balancing on their beaks. Now, you should be able to understand how toy works.

**Example 7:** At t = 0 a ball of mass M is traveling at speed V in the +x direction inside a box of mass M and length 2L which is at rest as shown below. Graph as a function of time a) the position of the ball; b) the position of the box; and c) the position of the center of mass.