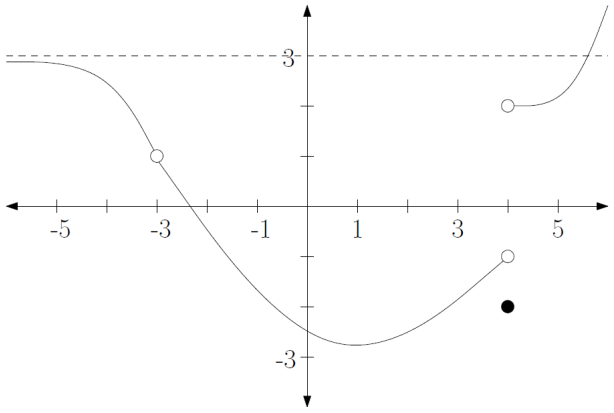
Calculus I Final Exam

Name: _____

Calculators are permitted. Please show your work.

1. Based on the graph of f given below, find the following.



(a) $f(-3)$

(b) $\lim_{x\to -3^-} f(x)$

(c) $\lim_{x\to -3^+} f(x)$

(d) $\lim_{x\to -3} f(x)$

(e) $f(4)$

(f) $\lim_{x\to 4^-} f(x)$

(g) $\lim_{x\to 4^+} f(x)$

(h) $\lim_{x\to 4} f(x)$

(i) $\lim_{x\to \infty} f(x)$

(j) $\lim_{x\to -\infty} f(x)$

2. If $f(x) = \frac{1}{x}$ find $f'(5)$ using the definition of derivative (using limits).

3. Find $\lim_{x \to 6^+} \frac{1-x}{x^2-6x}$.

4. Find the vertical and horizontal asymptotes of $f(x) = \frac{x-2}{x^2+x-6}$.

5. Suppose $f(x) = \frac{3e^{4x}}{4x^2-3x-1}$. Find $f(x)$, for $x = 0.1$, 0.01, and 0.001, and use this information to estimate $\lim_{x\to 0} f(x)$.

6. Find $\frac{d}{dx}\ 7x^4 - 10\sqrt{x} + e^x + \pi^8 - 6\sec x$.

7. If $f(x) = x^3 \sin(x) + 5$, find the equation of the tangent line to the graph of f at $x = 0$.

8. Find the derivative of $f(x) = \frac{\ln|x|}{3-\cos(x)}$.

9. A particle's initial position is $s(0) = 10$, and its velocity at time t is $v(t) = 20 - 10t$. Find $s(t)$, the position at time t, and $a(t)$, the acceleration at time t.

10. Air is blown into a spherical balloon at a rate of 700 cm^3 per minute. How fast is the radius of the balloon increasing when the volume is 400 cm^3?

11. Suppose $f(x) = x^3 + 6x^2 - 15x + 1$.

(a) Find all open intervals where f is increasing, all open intervals where f is decreasing, and all local maxima and minima of f.

(b) Find all open intervals where f is concave up, all open intervals where f is concave down, and all inflection points of f.

(c) Sketch a graph of f that displays the above characteristics.

12. A box with an open top is constructed from 1000 cm^2 of cardboard. What is the maximum possible volume the box can have?

13. Find $\int 2x^4 - 10\sqrt{x} + \frac{2}{5x^4} + \pi^5 + 3\csc^2(x)\ dx$.

14. Find $\int_{\pi^2}^{4\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}}\ dx$.