Foundations for Functions

Activity: Mystery Liquids (Linear Functions)

TEKS: (2A.1) **Foundations for functions.** The student uses properties and attributes of functions and applies functions to problem situations.
The student is expected to:
(B) collect and organize data, make and interpret scatter plots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

Overview: This lesson is a data analysis lab in which students gather mass and volume data for two mystery liquids, oil and water, and then use the data to explore linear functions. Connections to science are an important part of this algebra lesson, which deals with density. Active learning and communication are aspects of the lesson that are modeled for teachers. Students discuss the physical meaning of the slopes and y-intercepts of the various lines they have created from their scatter plots.

Materials: Fish tank (or other large, clear container) full of water
1 can of Diet Coke
1 can of regular Coke

For each group of four:
Balance
Two graduated cylinders
Liquid A (water with red food coloring added)
Liquid B (oil)
Large-size graph paper
Red and blue marking pens
Meter sticks
Graphing calculator
Part 1: Collecting Data
Mystery Liquids Data Table
Part 2: Graphing Data Manually and Finding Equation
Part 3: Using a Calculator to Graph and Determine the Linear Model
Part 4: Interpreting the Data
Directions for Using the TI-82

Grouping: Groups of 4

Lesson:

Procedures	Notes
1. **Introductory Activity** Ask the students what they think will	Be sure to check this out before you

Procedures	Notes
happen when the can of Coke and the can of Diet Coke are dropped into the fish tank full of water. Will they sink or will they float? Place one can of each in a fish tank at the front of the classroom. Some students are surprised to find out that the Diet Coke will float while the regular Coke sinks.	actually do it for your students because sometimes this demonstration does not work! It helps to use cold water and warm soda. The goal of this activity is to spark interest and provide a reference point for a discussion about density, which will follow as the lesson develops. At the end of the demonstration explain what the students will be doing in the lab.
2. **Group Roles** Have the students work in groups of four. Discuss the group roles with the students. Each group should have a facilitator, a materials person, a time-keeper, and a reporter. Each person will serve as grapher for one of the four sets of data. The materials person should gather everything that the group needs including all the handouts.	The roles used in cooperative learning give each student a responsibility and a specific task in the group so that each member is involved and no member ends up doing all of the work.
3. **Data Collection** Students work in groups and follow instructions for collecting the data given on *Part 1: Collecting Data*. In each group, two students collect data using liquid A (water), and two students collect data using liquid B (oil). After recording both the mass and the volume on the *Mystery Liquids Data Table*, the students repeat this process until they have completed six trials. When the data has been recorded, the students complete columns 4 and 5 on the data table. At this point each group of four has two	Remind students to read the graduated cylinder from the bottom of the meniscus and to place the graduated cylinder on a level surface when the reading is made. To measure mass, students first weigh the graduated cylinder; then they add about two cubic centimeters of liquid and weigh the graduated cylinder and the liquid.

Procedures	Notes
data tables—one for liquid A and one for liquid B.	
4. **Data Patterns** Lead a discussion with the entire class to verify the data patterns and check the work of the groups. Have each group determine the median value for column 5 for liquid A and for liquid B, and list these on the board.	The students need to see that in column 4 the values decrease, and in column 5, the values are closer to a constant value. Students should also see that for liquid A, the values in column 5 are all close to 1, and for liquid B, the values in column 5 are all less than 1.
5. **Scatter plots** Students should create their scatter plots. Those working with liquid A should use a red pen, and those working with liquid B should use a blue pen, as they create their graphs.	Be sure to remind the students to plot all of the graphs on the same sheet of graph paper and to share the work, making sure that each student completes one scatter plot and line of best fit.
6. **Linear Equations** Students will need *Part 2: Graphing Manually and Finding Equation*. After they have created the scatter plots and drawn the lines of best fit, the groups need to determine the equations for the lines. Using the slope and the y-intercept, the students should write the equation for the line using the form $y = mx + b$, where m is the slope of the line, and b is the y-intercept. By the time the groups finish this part of the lab, they should have the four scatter plots and lines drawn on the graph paper and the equations for these lines recorded next to each line on the graph.	Linear functions are fundamental to the study of algebra. Students need practice in recognizing scatter plots that are linear; they need to be able to draw an eye-ball line of best-fit; and they need to be able to determine an equation for a line. The fundamental property of linear models—a constant rate of change—should be explored through tables, graphs, and symbolic representations. Students need to remember that they can calculate the slope by determining the change in the y-values divided by the change in the x-values for any two points on the line. Another point to highlight is the use of independent and dependent variables. The students need to understand that in this case, volume is the

Procedures	Notes
	independent variable and mass is the dependent variable. The independent variable, or the volume, is plotted on the x-axis, and the dependent variable of the mass is plotted on the y-axis. Using these terms should also help students make connections between mathematics and science.
7. **Scatter plots and Linear Equations Using the Calculator** Students will need *Part 3: Using a Calculator to Graph and Determine the Linear Model.* They also may find it helpful to use *Directions for Using the TI-82.* Each student should enter and graph all of the data. The data for liquid A from columns 1, 2, and 3 of the data table should be entered into L1, L2, and L3; and the data for liquid B from columns 1, 2, and 3 should be entered into L4, L5, and L6. Each student should graph the scatter plot and use the calculator to determine the linear equation for the data sets: (L1, L2), (L1, L3), (L4, L5), and (L4, L6). Students should compare their answers and record the linear regression equations determined by the calculator on the graph paper next to the corresponding lines.	Many students may have noticed that on the graphing calculator there are two forms of the linear regression model, LinReg(ax+b) and LinReg(a+bx). The first model is similar to $y = mx + b$, but the second might be a more helpful model for some students. If students graph $y = a + bx$, they start at the y-intercept a, and then move over 1 unit to the right and then b units in the vertical direction. These two forms use both the slope and the y-intercept, which are fundamental concepts dealt with in linear functions. This lesson helps students understand that there is a relationship between those characteristics of the algebraic model and the physical situation that the model represents. Remind the students to round off to two decimal places. Students should verify that the linear models they calculated by hand are close to the equations determined by the calculator.

Procedures	Notes
8. **Data Interpretation** Provide each student a copy of *Part 4: Interpreting the Data.* Have each group display their graphs and linear equations. Students should discuss the answers to the interpretation questions within their groups. Pull the groups together for a class discussion.	Students should notice from their graphs that the two red lines are parallel to each other, and that the two blue lines are parallel to each other, with the slopes of the red lines being greater than the slopes of the blue lines. It should be emphasized and brought out in the discussion that the slope of the line is the density of the liquid. Students should also realize that the y-intercept is the mass of the graduated cylinder. From the graphs, students can see that the density of liquid A is greater than the density of liquid B. Have the students pour the two liquids together and discuss what they see. Point out that mathematically they determined that liquid B was less dense than liquid A, thus verifying what was determined mathematically.
9. **Calculating correlation** In their groups, have students predict what kind of correlation there will be; students should have a clear argument for their predictions. Then have each group calculate the correlation and compare it to their predictions. If their predictions were wrong, have them write an explanation of where they were mistaken and how to correct it.	Students may need to be shown how to calculate the correlation coefficient on the calculator. Also, this is a good place to discuss what the correlation coefficient measures.
10. **Back to the Beginning** Discuss what the results of the Mystery Liquids experiment have to	Students should be able to conclude that the density of Diet Coke is less

Procedures	Notes
do with the Coke/Diet Coke demonstration.	than one and therefore the slope of a Diet Coke line would be less steep than the line for liquid A. They should also be able to conclude that the slope for a Coke is greater than one and would be steeper than the line for liquid A.

Assessment: Throughout this lesson there are opportunities to gain valuable insight into student understanding. What do the students know about density? What do they bring to the opening demonstration? Group work gives the teacher the opportunity to move from group to group to evaluate the general level of success that the students are having at a variety of different points in the lesson.

How well do the students do with data collection? Do they understand how to fill in the data chart and make the required calculations? When students report the ratio of mass to volume for liquid A and liquid B, not only is the teacher able to quickly determine where the class is at that point, but also the students are able to determine where they are compared to the rest of the class.

Are the students able to make the scatter plots and determine the linear equations, both by hand and with the graphing calculator? Assess these skills by watching the students as they work in groups, and also ask questions of the students both to direct their thinking and to evaluate their understanding.

Group reports toward the end of the class give the students the opportunity to share their work both visually and orally with the entire class. Again, the teacher can evaluate students' understanding while the students have the opportunity for self-assessment. Pulling the lesson together by making sure that students discuss the key issues is a powerful means of assessing student understanding.

Extensions: This lesson does not depend on the graphing calculator, although use of the calculator does provide nice practice and a good check on the work the students do by hand. Item 7 in the procedure could be skipped.

While appropriate for any class studying linear models for the first time, this lesson would provide thoughtful review for a more advanced class.

The concept of density is an important one for science students. Check with your science teachers for other sets of density data, or have students collect data and calculate the density of other materials.

An extension for the students: Direct variation is described as a situation in which *y* varies directly as *x* or where *y* is directly proportional to *x*, and is expressed as $y = kx$ where *k* is any non zero number. Do any of the lines on the graphs illustrate this relationship? Defend your position.

While you might be inclined to try calculating the densities of Coke and Diet Coke, attempts to do this have not been very successful. The problem is that the densities of Coke and Diet Coke are both very close to 1 and precise measurements are difficult to make.

Are all linear functions direct proportions? Some students are under the misconception that this is true. A direct proportion, for example when *y* varies directly as *x*, is represented by the equation $y = kx$, where *k* is the constant of proportionality. Direct proportion is an important concept in both mathematics and science classes.

Suggested Resources:

Source: Mystery Liquids is an original lesson written by Elaine Wizda and Kathleen Thompson
http://www.pbs.org/mathline

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades 9 - 12 : Algebra in a Technological World*. Reston, Virginia: National Council of Teachers of Mathematics, 1991.

Internet location:
http://www.mtse.uiuc.edu/regression/titlepage.html
This is an excellent site that gives step-by-step instructions for finding a linear regression equation. It also has two data files that can be downloaded.

Internet location:
http://www.glenbrook.k12.il.us/gbsmat/travel/bonvoyage.html
Currency exchange rates, mathematics, and the Internet are woven together to allow students to investigate currency exchange rates of different countries and create a report. Student handouts, grading rubric, and Internet links are included.

Internet location:
http://www.cs.rice.edu/~sboone/Lessons/Titles/lphouse.html
This site has students collect housing prices over the Internet and then develop a linear model for the relationship between the cost of

the house and the number of square feet. From there, students will interpolate and extrapolate the cost or size of any house.

Internet location:
http://www.cs.rice.edu/~louviere/Lessons/les6.html
"It's a *Matter* of Density" is a lesson that will allow students to further explore the concept of density. The lesson is complete with data tables, questions, links, and a scoring rubric.

Internet location:
http://www.glenbrook.k12.il.us/gbsmat/regress/regress.html
This Internet project leads students through determining a linear regression model on the TI-82 calculator.

Internet location: **http://www.math.psu.edu/OtherMath.html**
This site offers a collection of resources from Penn State University.

Part 1: Collecting Data

In your group of four, two students will collect data on liquid A, and two will collect data on liquid B. Share your data within the group to complete both data tables. When measuring, have your partner check your measurements.

1. Find the mass of an empty graduated cylinder.
   Record the mass to the nearest 0.01 gm at the top of the data table.

2. Half of your group should collect and record data in a data table for liquid A, and the other half should collect and record data in a separate data table for liquid B. Record which liquid you are analyzing at the top of the data table. Pour about 2 cm^3 (ml) of your liquid into the graduated cylinder. Record the exact volume of the liquid in column 1 (trial 1) in the data table (to the nearest 0.1).

3. Find the mass of the system (graduated cylinder containing the liquid). Record this mass in column 2 of your data table. Find the mass of the liquid alone and record this value in column 3 of your data table.

4. Add about 2 additional cm^3 of the same liquid to the graduated cylinder. Again record the exact volume of the liquid now in the graduated cylinder in column 1 (trial 2). Repeat step 3 and again record both masses in columns 2 and 3.

5. Repeat step 4 until you have completed six trials. Record the data after each trial.

6. Complete columns 4 and 5 of the data table.

When you have completed collecting data, you should have two data tables completed in your group—one for liquid A and one for liquid B.

Mystery Liquids

Data Table for Liquid _____

(specify liquid A or B)

Mass of empty graduated cylinder __________

Trial	Column 1 Volume of Liquid	Column 2 Mass of Cylinder plus Liquid	Column 3 Mass of Liquid	Column 4 Ratio of Column 2 to Column 1	Column 5 Ratio of Column 3 to Column 1
1					
2					
3					
4					
5					
6					

Part 2: Graphing Data Manually and Finding Equation

On one large sheet of graph paper, graph the following sets of data:

From Data Table for Liquid A:

1. Using a red pen, graph data from column 1 and column 2 for liquid A. Which column should represent the independent axis? Which one are you controlling? Draw an approximate best-fit line. Find the equation of this line algebraically. Show your work below.

2. Using a red pen, graph data from column 1 and column 3 for liquid A. Draw an approximate best-fit line. Find the equation of the line algebraically. Show your work below.

From Data Table for Liquid B:

3. On the same sheet of graph paper that you used for Liquid A, use a blue pen and graph data from column 1 and column 2 for liquid B. Which column should represent the independent axis? Which one are you controlling? Draw an approximate best-fit line. Find the equation of the line algebraically. Show your work below.

4. Using a blue pen, graph data from column 1 and column 3 for liquid B. Draw an approximate best-fit line. Find the equation of the line algebraically. Show your work below.

Part 3: Using a Calculator to Graph and Determine the Linear Model

Each person should enter and graph all data.

Liquid A

1. Enter the data from the data table for liquid A into L1, L2, and L3.

2. Graph a scatter plot of data from columns 1 and 2. (L1, L2).

3. Determine the best-fit line by calculating the linear regression model for the (L1, L2) data. Graph the line on the calculator. How good a fit is the line? Record the linear equation on your graph paper. Round values to two decimal places.

4. Turn off all scatter plots and equations. Graph a scatter plot of the data from columns 1 and 3. (L1, L3).

5. Determine the best-fit line by calculating the linear regression model for the (L1, L3) data. Graph the line on the calculator. How good a fit is the line? Record the linear equation on your graph paper. Round values to two decimal places.

Liquid B

1. Enter the data from the data table for liquid B into L4, L5, and L6.

2. Turn off all scatter plots and equations. Graph a scatter plot of the data from columns 1 and 2. (L4, L5).

3. Determine the best-fit line by calculating the linear regression model for (L4, L5) data. Graph the line on the calculator. How good a fit is the line? Record the linear equation on your graph paper. Round values to two decimal places.

4. Turn off all scatter plots and equations. Graph a scatter plot of the data from columns 1 and 3. (L4, L6).

5. Determine the best-fit line by calculating the linear regression model for the (L4, L6) data. Graph the line on the calculator. How good a fit does it seem to be? Record the linear equation on your graph paper. Round values to two decimal places.

Part 4: Interpreting the Data

1. Compare the graphs for liquid A with the graphs for liquid B. How are they similar? How are they different?

2. Compare the slopes of the two lines for liquid A. Do the same for liquid B. What could be the reason for your observation?

3. Compare the slopes representing liquid A with the slopes representing liquid B. What could be the reason for your observation?

4. Explain the physical meaning of the y-intercepts.

5. What does the slope represent?

Directions for Using the TI-82 to...
Graph a Scatter plot; Determine a Linear Model; and Graph the Model

To enter data:

STAT 1 : EDIT ENTER

(Clear all lists by using the up arrow button to go to the top and press clear.)

Enter data using L1 as the independent variable and L2 as the dependent variable.

To graph a scatter plot:

2nd Y (STAT PLOT) 1 : PLOT1 ENTER

Plot on; type: scatter

X list: L1; Y list: L2; Mark: 1st symbol.

To choose window and graph:

ZOOM 9 : ZoomStat ENTER

YOU SHOULD SEE A SCATTER PLOT.

To find linear regression:

STAT CALC 5 : LinReg(ax+b) ENTER

To graph the line:

Y = VARS 5 : Statistics EQ 7 : RegEQ GRAPH

YOU SHOULD GET A LINE THOUGH THE SCATTER PLOT.

Selected Answers

Part 4: Interpreting the Data

1. Compare the graphs for liquid A with the graphs for liquid B. How are they similar? How are they different?

   The graphs for liquids A and B are all straight lines. The pairs of lines have different slopes, but the same y-intercept.

2. Compare the slopes of the two lines for liquid A. Do the same for liquid B. What could be the reason for your observation?

   Both A lines have the same slope and both B lines have the same slope. This makes sense because the slope of the line represents the change in mass divided by the change in volume. In one case, the mass of the graduated cylinder is included, and in the other case this mass has been subtracted.

3. Compare the slopes representing liquid A with the slopes representing liquid B. What could be the reason for your observation?

   The slope of the A lines is about 1, and the slope of the B lines is less than 1. This is because liquid A (water) has a density of 1, and liquid B (cooking oil) has a density that is less than 1.

4. Explain the physical meaning of the *y*-intercepts.

   The y-intercept is the mass of the graduated cylinder. If there is no liquid in the graduated cylinder, then the y-intercept gives the mass.

5. What, if anything, does this lab have to do with density?

   The slope of the lines represents the densities of the liquids. Density is mass divided by volume.