Number, Operation, and Quantitative Reasoning

Activity: Problem Solving with Fractions

TEKS: (6.2) **Number, operation, and quantitative reasoning.** The student adds, subtracts, multiplies, and divides to solve problems and justify solutions.

The student is expected to:

(A) model addition and subtraction situations involving fractions with objects, pictures, words, and numbers;

(6.3) **Patterns, relationships, and algebraic thinking.** The student solves problems involving direct proportional relationships.

The student is expected to:

(B) represent ratios and percents with concrete models, fractions, and decimals; and

(6.11) **Underlying processes and mathematical tools.** The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school.

The student is expected to:

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.

Overview: Students will discover a means for solving problems involving fractions by focusing on using a complete definition for fractions. Students will develop their abilities in the use of diagrams as a problem-solving strategy.

Materials: *The Definition for Fractions* (Sheet 1)
*Problem Solving with Fractions* (Transparency 1)
*Example* (Transparency 2)
*Additional Practice Problems*
*Problem Solving with Fractions* (Sheets 2, 3, and 4)
**Grouping:** Small groups

**Time:** 2 or 3 class periods

### Lesson:

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<td><strong>1.</strong> Distribute <em>The Definition for Fractions</em> (Sheet 1) and use a transparency of this sheet to introduce the three-part definition of a fraction. It is important that the students use this definition during their work in the following activities.</td>
<td>Asking students to point out each part of the definition whenever a fraction is being discussed is helpful. For example, when a picture such as the rectangle shown on sheet one is being considered, many different answers can be justified as being correct through a consideration of different “1’s.” The student can use the three-part definition to show how the fraction $\frac{3}{4}$, $\frac{3}{5}$, and $\frac{3}{2}$ could all be illustrated by choosing the appropriate “1” for each answer. In the instance of $\frac{3}{4}$ for example, the “1” is the complete drawing with an empty square in the middle; $\frac{3}{5}$ would have “1” of five equally large squares such that one of the squares is placed over the other four, partially obscuring portions of them; $\frac{3}{2}$ could be justified by considering a “1” that consists of the lower two L-shaped parts of the square picture frame.</td>
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<p>| <strong>2.</strong> After discussing Sheet 1, introduce the use of this definition and pictorial models for fractions by showing <em>Problem Solving with Fractions</em> (Transparency 1). | The transparency is designed to have the right half covered and revealed step by step as the process is applied. Students would probably record just one diagram for this problem, as shown in figure 1. |</p>
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| 3. **Additional Practice Problems** will provide more practice for the students and can be discussed until students are able to solve the problems on sheet 2. | Several other problems are offered here: 
1. $45 = \frac{5}{9}$ the cost of ski boots. 
   What is the cost of the ski boots? ($81.00)
2. $48 = \frac{6}{11}$ the number of marbles in the bag. How many marbles are in the bag? (88)
3. $213 = \frac{3}{5}$ the number of girls in the school. How many girls are in the school? (355)
4. $4.36 = \frac{4}{11}$ cost of gloves. What is the cost of the gloves? ($11.99) |
<p>| 4. Leave transparency 1 on the overhead as a reference for students while they are working on Problem Solving with Fractions (Sheet 2). | |
| 6. <strong>Example</strong> (Transparency 2) is designed to introduce the students to the type of modeling with fractions that is useful when the “whole” evolves during the representation of its parts. Use this transparency to prepare the students for doing Problem Solving with Fractions (Sheet 3). Leaving the transparency projected on the screen during this period may be helpful. | |
| 7. The second question in problem 3 on Problem Solving with Fractions (Sheet 3) introduces the combination of processes from the previous activity sheets that are needed for Problem Solving with Fractions (sheet 4). | After discussing this question, one can consider a second example by returning to the problem on transparency 2. Ask the students how they would solve this problem if the “whole” object is a twenty-eight-inch sandwich from Subway and we wanted to know how long each of the three pieces should be. It would be a good idea to review |</p>
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<td>the process in its entirety with the additional information before assigning work on sheet 4.</td>
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<td>8. Journal Writing: Have students describe in their own words the problem solving process when working with fractions.</td>
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**Homework:** Have students create a problem like the ones done in class along with the answer key.
Problem Solving with Fractions

Suppose I have $27 in my pocket and this amount represents $\frac{3}{4}$ of the cost of an item. How much does this item cost?

-Solution process-

A. Represent and Label the whole

B. Divide the whole into equal parts

C. Shade and label the number of equal parts of interest

Total cost of the item

Total cost of item

Total cost of item

$27

Transparency 1
Example

A whole object has been broken into separate pieces. The largest piece is 4 times as large as the smallest pieces, and the middle-size piece is 2 times the size of the smallest piece. What fraction can be used to describe each of the 3 pieces in terms of the original whole unit?

Solution Process

a) Represent the parts separately showing given relationships.

b) What represents the whole? (The complete set of 3 pieces.)

c) Into how many equal pieces is the whole divided? (7)

d) Use fractions to label the part of the whole represented by each pieces.

\[
\frac{4}{7}, \frac{1}{7}, \frac{2}{7}
\]
Additional Practice Problems

Carlos has ridden his bicycle 110 miles this week. This represents $\frac{5}{8}$ of the total number of miles that he wants to ride. Let’s find the total number of miles he plans to ride.

If $2.55 is $\frac{3}{5}$ the cost of a soccer ball, how much does the soccer ball cost?

Ray has read $\frac{5}{7}$ of a book; he has read 105 pages. How many pages does this book have?
The Definition for Fractions
Sheet 1

1. To have a fraction, one needs to have three things:
   Something has to be designated as a “whole” thing. We usually give the whole thing a value of “1.”

2. The whole thing is divided into a certain number of equal pieces. The number of pieces the whole is divided into is designated by the denominator.

3. A certain number of these equal pieces is emphasis or considered. The number of pieces being considered is designated by the numerator.

Example (Using Transparency #1):
1. What fraction is illustrated in this picture? ____________

2. Use the definition of fractions to explain your answer.
   a) What is the whole? ________________________________

   b) Into how many equal parts is the whole divided? ___

   c) How many of these parts are shaded? _____________

3. Are any other fractions illustrated? (Hint: use a different whole.)
Problem Solving with Fractions
Sheet 2

Use the definitions and approach we’ve been using in class for fractions to solve the following problems. For each problem, draw a picture to show what fraction you used.

1. If $1.95 is \frac{3}{4} of the cost of a cheeseburger, how much does a cheeseburger cost?

2. If $3,575.00 is \frac{5}{7} the price of a car, how much does the car cost?

3. Mike has read \frac{4}{9} of a book; he has read 1,089 pages. The book has how many pages?

4. Mary has ridden her bike 261 miles, which represents \frac{3}{8} of the distance from her house to San Francisco. How far does she live from San Francisco?
Problem Solving with Fractions
Sheet 3

Use the definitions and approach we’ve been using in class for fractions to solve the following problems. For each problem, draw a picture to show what fraction you used.

1. A whole object has been broken into 4 pieces, all of different sizes. Each piece is 2 times the next smaller piece. What fractions describe each piece of the whole object?

2. A collection of marbles has been divided into 3 different sets. The middle-sized set is 2 times the size of the smallest set, and the largest set is 3 times as large as the middle-sized set. What fraction describes each part of the total marble collection?

3. Mr. Jones drove to Boise, Idaho, in 4 days. On Monday and Wednesday he traveled exactly the same distance. On Tuesday he traveled 2 times as far as he did on Monday, and on Thursday he traveled 3 times as far as he did on Wednesday. Which fraction describes the part of the trip covered each day? If the total trip covered 602 miles, how far did Mr. Jones travel each day of his trip?
Problem Solving with Fractions
Sheet 4

1. Mrs. Smith rode the bus 720 miles in 3 days. On the first day, she traveled 3 times as far as she did on the second day. On the third day, she traveled 2 times as far as she did on the second day. How far did she travel each day?

_______ 1st day
_______ 2nd day
_______ 3rd day

2. The neighborhood grocery store sold 1,463 bottles of soft drinks last month. Twice as many bottles of root beer were sold than lemon-lime soda, and twice as many bottles of cola were sold than root beer. How many bottles of each type of soft drinks were sold?

_______ Lemon-lime
_______ Root beer
_______ Cola

3. A total of 960 students attend Bosco School. Some students walk to school, some ride the bus, and the rest come by car. The number that rides the bus is 6 times greater than the number that comes by car. The number that walks to school is ½ the number that rides the bus. How many students come to school by each form of transportation?

_______ Car
_______ Bus
_______ Walk
Carlos has ridden his bicycle 110 miles this week. This represents \(\frac{5}{8}\) of the total number of miles that he wants to ride. Let’s find the total number of miles he plans to ride.

\[
\begin{array}{c|c|c|c}
110 \text{ miles} & 22 & 22 & 22 \\
\end{array}
\]

Total number of miles is 176

If \(2.55\) is \(\frac{3}{5}\) the cost of a soccer ball, how much does the soccer ball cost?

\[
\begin{array}{c|c|c|c}
2.55 & .85 & .85 \\
\end{array}
\]

Total cost = \$4.25

Ray has read \(\frac{5}{7}\) of a book; he has read 105 pages. How many pages does this book have?

\[
\begin{array}{c|c|c|c|c|c|c}
21 & 21 & & & & \\
\end{array}
\]

Total number of pages: 147
Problem Solving with Fractions (Answers)
Sheet 2

Use the definitions and approach we’ve been using in class for fractions to solve the following problems. For each problem, draw a picture to show what fraction you used.

1. If $1.95 is \frac{3}{4} of the cost of a cheeseburger, how much does a cheeseburger cost?

\[
\begin{align*}
\text{4 Parts} \\
\$1.95 \quad 0.65 \quad \text{Total cost of cheeseburger} = $2.60
\end{align*}
\]

2. If $3,575.00 is \frac{5}{7} the price of a car, how much does the car cost?

\[
\begin{align*}
\text{7 parts} \\
\$3,575 \quad 715 \quad 715 \quad \text{price of car} = $5,005
\end{align*}
\]

3. Mike has read \(\frac{4}{9}\) of a book; he has read 108 pages. The book has how many pages?

\[
\begin{align*}
\text{9 parts} \\
108 \text{ pages} \quad 135 \text{ pages} \quad (5 \times 27) = 243 \text{ pages}
\end{align*}
\]

4. Mary has ridden her bike 261 miles, which represents \(\frac{3}{8}\) of the distance from her house to San Francisco. How far does she live from San Francisco?

\[
\begin{align*}
261 \text{ miles} \quad 261 \text{ miles} \quad 87 \quad 87 \quad = 696 \text{ miles}
\end{align*}
\]
Problem Solving with Fractions (Answers)
Sheet 3

Use the definitions and approach we’ve been using in class for fractions to solve the following problems. For each problem, draw a picture to show what fraction you used.

1. A whole object has been broken into 4 pieces, all of different sizes. Each piece is 2 times the next smaller piece. What fractions describe each piece of the whole object?

   \[
   \frac{1}{15} \quad \quad \frac{2}{15} \quad \quad \frac{1}{15} \quad \quad \frac{8}{15}
   \]

2. A collection of marbles has been divided into 3 different sets. The middle-sized set is 2 times the size of the smallest set, and the largest set is 3 times as large as the middle-sized set. What fraction describes each part of the total marble collection?

   \[
   \frac{1}{9} \quad \quad \frac{2}{9} \quad \quad \frac{6}{9}
   \]
3. Mr. Jones drove to Boise, Idaho, in 4 days. On Monday and Wednesday he traveled exactly the same distance. On Tuesday he traveled 2 times as far as he did on Monday, and on Thursday he traveled 3 times as far as he did on Wednesday. Which fraction describes the part of the trip covered each day?

\[
M = \frac{86}{7} = \frac{1}{7} = 86 \text{ mi.} \\
W = \frac{86}{7} = \frac{1}{7} = 86 \text{ mi.} \\
T = \frac{86 \times 2}{7} = \frac{172}{7} = 172 \text{ mi.} \\
\text{Th} = \frac{86 \times 3}{7} = \frac{258}{7} = 258 \text{ mi.}
\]

Total = 602 miles.

If the total trip covered 602 miles, how far did Mr. Jones travel each day of his trip?

- Monday 86 miles
- Tuesday 172 miles
- Wednesday 86 miles
- Thursday 258 miles