1. The only force acting on a 1.9 kg body as it moves along the x axis varies as shown below. The velocity of the body at \( x = 0 \) is 4.0 m/s.

\[ F_x (N) \]

\[ \begin{array}{c|c|c|c|c}
\text{Find} & 0 & 2 & 3 & 4 & 5 \\
\hline
\text{x (m)} & 4 & & & & \\
\end{array} \]

A. What is the initial kinetic energy of the body?

\[ K = \frac{1}{2} M v^2 = \frac{1}{2} (1.9 \text{kg}) (4 \text{m/s})^2 \]

\[ K = 15.2 \text{J} \]

B. How much work was done on the body as the block was displaced 4 m?

Work is the area under a \( F \times x \) graph.

\[ W = \frac{1}{2} (4N)(1m) + \frac{1}{2} (1m)(-4N) + (2m)(-4N) \]

\[ W = -8 \text{J} \]

Section 6.2 Text
C. What is the maximum kinetic energy obtained by the body?

\[ W = K_f - K_i \]
\[ K_f = W + K_i = W + 15.2J \]

Find Max Work \( \Rightarrow \) Max Area under curve
\[ W = \frac{1}{2} \times (1m)(4N) = 2J \]
\[ K_f = 2J + 15.2J = 17.2J = \frac{1}{2}mv_f^2 \]

2. A 260 g block is dropped onto a relaxed vertical spring that has a spring constant of \( k = 2.1 \text{ N/cm} \). The block becomes attached to the spring and compresses the spring 14 cm before momentarily stopping.

\[ k = \frac{210N}{m} \]
\[ k = 2.1 \text{ N/cm} \left( \frac{100 \text{ cm}}{1 \text{ m}} \right) \]

A. While the spring is being compressed, what work is done on the block by the gravitational force on it?

Need Section 6-4 of Textbook
\[ W_g = -\Delta U_g = -(U_{gf} - U_{gi}) = -U_{gf} + U_{gi} \]
\[ W_g = -mg_{y_f} + mg_{y_i} \]
\[ W_g = -(0.26 \text{ kg})(9.8 \text{ m/s}^2)(-0.14 \text{ m}) \]
\[ W_g = 0.3575 \text{ J} \]
B. What work is done on the block by the spring force while the spring is being compressed?

\[ W_s = -\Delta U_s = -\frac{1}{2} k x_f^2 + \frac{1}{2} k x_c^2 \]

\[ W_s = -\frac{1}{2} k x_f^2 = -\frac{1}{2} \left( 210 \frac{N}{m} \right) (-0.14)^2 \]

\[ W_s = -2.0585 \text{ J} \]

C. What is the speed of the block just before it hits the spring? (Assume that friction is negligible.)

Method #1:

\[ W_{mc} = E_f - E_i \]

\[ E_f = E_i \]

\[ \frac{1}{2} k x_f^2 + mg y_f = \frac{1}{2} m v_i^2 \]

\[ W_{net} = \Delta K \]

\[ W_g + W_s = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \]

\[ v_i^2 = -\frac{2}{m} (W_g + W_s) = -\frac{2}{0.26 \text{ kg}} \left( 0.357 \text{ J} - 2.0585 \text{ J} \right) \]

\[ v_i = 3.62 \text{ m/s} \]
3. Show that a block released from rest at the top of a frictionless hemisphere of radius R will leave the surface at an angle of \( \theta = \cos^{-1}\left(\frac{2}{3}\right) \) as measured with respect to the vertical.

\[ \text{Applying Energy Analysis} \]
\[ W_{nc} = \Delta E \]

The only force doing work on the block is gravity which is conservative.

\[ \Rightarrow W_{nc}^{0} = E_f - E_i \]
\[ E_f = E_i \]
\[ K_f + U_f = K_i + U_i \text{ as } U_i = 0 \]
\[ \frac{1}{2} m v^2 + m g R \cos \theta = m g R \]

(1) \[ \frac{v^2}{2} + g R \cos \theta = g R \]

To find \( v \), we apply our knowledge of Ch. 4 & 5

Free body diagram of block
\[ \Sigma F_y = m a_y \]
\[ N - w \cos \theta = m \left(\frac{-v^2}{R}\right) \]

At point where block leaves surface, \( N = 0 \).

\[ \Rightarrow - m g \cos \theta = - \frac{m v^2}{R} \]

\[ \Rightarrow \boxed{v^2 = R g \cos \theta} \]
Substituting \( \Theta \) into \( \theta \) gives us

\[
\frac{1}{2} Kg \cos \Theta + Kg \cos \Theta = Kg
\]

\[
\frac{1}{2} \cos \Theta + \cos \Theta = 1
\]

\[
\frac{3}{2} \cos \Theta = 1
\]

\[
\cos \Theta = \frac{2}{3}
\]

\[
\Theta = \cos^{-1} \left( \frac{2}{3} \right)
\]
4. In the figure below, a small block of mass \( m = 0.033 \text{ kg} \) can slide along the frictionless loop-the-loop. The block is released from rest at point \( P \), at height \( h = 5R \) above the bottom of the loop. (The height of the loop is \( R = 30 \text{ cm} \).)

What is the magnitude of the horizontal component of the net force acting on the block at point \( Q \)?

![Diagram of loop-the-loop with point Q marked]

From Newton's Law (Ch. 4.5), we have:

\[
\begin{align*}
&\mathbf{F}_x = ma_x \\
-N = m\left(-\frac{U^2}{R}\right) \\
\Rightarrow &\quad N = \frac{mU^2}{R}
\end{align*}
\]

To find \( U \), we need to apply Energy Analysis:

\[
W_{nc} = E_f - E_i
\]

But the only force doing work is gravity which is conservative.

\[
K_f + U_f = K_i + U_i
\]

\( U_i = 0 \) since at rest

\[
\frac{1}{2}mU^2 + mgR = mg(5R)
\]

\[
\frac{1}{2}mU^2 = mg(4R)
\]

\( U^2 = 8gR \)

Substituting into \( N \), we have:

\[
N = \frac{m(8gR)}{R} = 8mg = 8W \quad \text{"8 times the weight!"}
\]

\[
N = 8(0.033\text{kg})(9.8\text{m/s}^2) \approx 2.59N
\]
5. A 9.40 kg block in the figure below is from rest at point A with a speed of 4.3 m/s down the hill. The track is frictionless until point C. The coefficient of kinetic friction between the track and the mass is 0.150 in the region between points C and D. The track is again frictionless after point D. The distance between points C and D is 1.75 m. Assume that the block maintains contact with the track.

A. What is the magnitude of the velocity of the mass at point B?

\[
W_{net} = E_B - E_A
\]

\[
E_B = E_A
\]

\[
\frac{1}{2} m v_B^2 + mgh_B = \frac{1}{2} m v_A^2 + mgh_A
\]

\[
\frac{1}{2} m v_B^2 = \frac{1}{2} m v_A^2 + mg(h_A-h_B)
\]

\[
v_B^2 = v_A^2 + 2g(h_A-h_B)
\]

\[
v_B = \sqrt{v_A^2 + 2g(h_A-h_B)}
\]

\[
v_B \approx \sqrt{(4.3m/s)^2 + 2(8.8m/s^2)(1.75m)} \approx 3.98 m/s
\]

B. How much work is done by the normal force along the path between points A and B?

\[
\vec{N} \perp \Delta \vec{r} \text{ along entire path so the normal force does } No \text{ work !!!}
\]

\[
W_N = 0 \text{ J}
\]
C. Calculate the work due to friction as the block moves from point C to point D.

\[ \mathbf{f} = \mu N \]

Applying Newton II, we have

\[ \Sigma F_y = M a^y \]
\[ N - W = 0 \]
\[ N = W = mg \]

\[ \mathbf{f} = \mu mg \]

\[ W = \mathbf{f} \cdot \Delta \mathbf{r} = \mathbf{f} \Delta r \cos (180°) = -\mu mg \Delta r \]

\[ W = -(0.150)(9.40\text{kg})(9.8\text{m/s}^2)(1.75\text{m}) \]
\[ W \approx -24.2 \text{ J} \]

D. At point D, the block hits a spring with a spring constant of 182 N/m. Calculate the maximum distance the spring is compressed.

\[ W_{nc} = E_f - E_A \]
\[ W_{nc} = (K_f + U_f) - (K_A + U_A) \]
\[ W_{nc} = \frac{1}{2} k x^2 - \left( \frac{1}{2} m v_d^2 + mg y_A \right) \]

\[ \frac{1}{2} k x^2 = W_{nc} + \frac{1}{2} m v_d^2 + mg y_A \]

\[ x^2 = \frac{2W_{nc} + m v_d^2 + 2mg y_A}{k} \]

\[ x = \sqrt{\frac{2W_{nc} + m v_d^2 + 2mg y_A}{k}} \]

But \( W_{nc} \) is the work by friction between C\&D

\[ x \approx \sqrt{\frac{2(-24.2 \text{ J}) + (9.40\text{kg})(9.8\text{m/s}^2)^2}{182 \text{ N/m}}} \]
\[ x \approx 3.21 \text{ m} \]
E. What is the net work done by all forces on the block as it moves from point A till it comes to rest with the spring compressed?

\[ W_{net} = K_f - K_A \]

\[ W_{net} = -\frac{1}{2} m v_A^2 \]

\[ W_{net} = -\frac{1}{2} (9.40 \text{ kg})(4.3 \text{ m/s})^2 \]

\[ W_{net} \approx -86.9 \text{ J} \]