Probability

I. Definitions

a. An **event** is any collection of results or outcomes of a procedure.
b. A **simple event** is an outcome or an event that cannot be further broken down into smaller components.
c. The **sample space** for a procedure consists of all possible simple events.
d. The **complement** of event *A*, denoted by $\bar{A}$, consists of all outcomes in which event *A* does not occur.
e. A **compound event** is any event combining two or more simple events.
f. Events *A* and *B* are **mutually exclusive** if they cannot occur at the same time.
g. Two events *A* and *B* are **independent** if the occurrence of one does not affect the probability of the occurrence of the other.
h. If *A* and *B* are not independent, they are said to be **dependent**.
i. A **conditional probability** of an event is a probability obtained with additional information that some other event has already occurred.

II. Notation for probabilities

a. ***P*** denotes a probability.
b. ***A*, *B*,** and ***C*** denote specific events.
c. ***P(A)*** denotes the probability of event *A* occurring.
d. ***P*($\bar{A}$)** denotes the probability of event *A* not occurring.
e. ***P*(*A* or *B*)** denotes the probability that event *A* or event *B* occur, but not at the same time.
f. ***P*(*A* and *B*)** denotes the probability that event *A* and event *B* both occur at the same time.
g. ***P*(*B* | *A*)** denotes the conditional probability of event B occurring after it is assumed that event A has already occurred.

III. Rules for simple events

a. Rule 1: Relative frequency approximation of probability

i. Conduct (or observe) a procedure, and count the number of times that event *A* actually occurs. Based on these actual results, *P(A)* is estimated as follows:

ii. $P(A) = \frac{\text{number of times } A \text{ occured}}{\text{number of times trial was repeated}}$

b. **Rule 2: Classical approach to probability** (requires equally likely outcomes)

i. Assume that a given procedure has n different simple events and that each of those simple events has an equal chance of occurring. If event *A* can occur in *s* of these *n* ways, then:

ii. $P(A) = \frac{\text{number of ways } A \text{ can occur}}{\text{number of different simple events}} = \frac{s}{n}$

c. Rule 3: Subjective probabilities

i. *P(A)*,the probability of event *A*, is estimated by using knowledge of the relevant circumstances.

IV. Rules for compound events

a. Addition rule

i. To find $P(A \text{ or } B)$, find the sum of the number of ways event A can occur and the number of ways B can occur, adding in such a way that every outcome is counted only once.

ii. $P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B)$

iii. If A and B are mutually exclusive, then the formula simplifies to:

1. $P(A \text{ or } B) = P(A) + P(B)$

b. Rule of conditional probability

i. The conditional probability of B given A can be found by assuming that A has already occurred and, working under that assumption, calculating the probability that event B will occur.

ii. $P(B \mid A) = \dfrac{P(A \text{ and } B)}{P(A)}$

c. Multiplication rule

i. When finding the probability that event A occurs in one trial and event B occurs in the next trial, multiply the probability of event A by the probability of event B, but be sure that the probability of event B takes into account the previous occurrence of event A.

ii. $P(A \text{ and } B) = P(A) * P(B \mid A)$

iii. If A and B are independent, then the formula simplifies to:

1. $P(A \text{ and } B) = P(A) * P(B)$

V. Bayes' theorem

a. With **Bayes' theorem**, we revise a probability value based on additional information that is later obtained. One key to understanding the essence of Bayes' theorem is to recognize that we are dealing with sequential events, whereby new additional information is obtained for a subsequent event, and that new information is used to revise the probability of the initial event.

b. A **prior probability** is an initial value originally obtained before additional information is obtained.

c. A **posterior probability** is a probability that has been revised using additional information that is later obtained.

d. According to Bayes' theorem, the probability of event A, given that event B has subsequently occurred, is

i. $P(A \mid B) = \dfrac{P(A) * P(B \mid A)}{[P(A) * P(B \mid A)] + [P(\bar{A}) * P(B \mid \bar{A})]}$

Example: Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

VI. Counting

a. Fundamental Counting Rule

i. For a sequence of two events in which the first event can occur *m* ways and the second event can occur *n* ways, the events together can occur a total of *m**n* ways.

b. Factorial Rule

i. A collection of n different items can be arranged in order *n!* different ways. This reflects the fact that the first item may be selected *n* different ways, the second item may be selected *n* – 1 ways, and so on.

c. Permutations Rule (when items are all different)

i. The number of permutations, or sequences, of *r* items selected from *n* available items without replacement is

1. $P = \frac{n!}{(n-r)!}$

d. Permutations Rule (when some items are identical to others)

i. If there are *n* items with n_1 alike, n_2 alike, …, n_k alike, the number of permutations of all n items is

1. $P = \frac{n!}{n_1!n_2!\ldots.n_k!}$

e. Combinations Rule

i. The number of combinations of *r* items from *n* different items is

1. $C = \frac{n!}{(n-r)!r!}$

ii. When different orderings of the same items are counted separately we have a permutation problem, but when different orderings of the same item are not counted separately we have a combination problem.