PROBABILITY

PROBABILITY = chance; calculated risk; estimated outcome.

P = probability that an event will happen

Q = probability that the event will NOT happen

$P + Q = 1$ $P = 1 - Q$ $Q = 1 - P$

LAWS:

1. The probability of a single event occurring that is one of a set of mutually exclusive events is the **SUM** of the probabilities of the single events.

Mutually Exclusive = occurrence of one event excludes the possibility of the other event.

CARDS: chance of drawing the ace of spades = 1/52

Chance of drawing an ace = 4/52 = 1/13

Chance of drawing a spade = 13/52 = 1/4

2. The probability of TWO OR MORE *independent* events occurring TOGETHER is the **PRODUCT(multiplication)** of each separate probability.

Independent = the occurrence of one event has NO influence on the occurrence of the other event(s).

Follows the mathematical formula for expanding the binomial $(a+b)^n$.

a = probability of one event; b = probability of 2nd event; n = # of replications

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

If 5 cows are bred, what is the probability of getting 5 bull calves?

a = bull; 5 replications; ➔ $a^5 = (1/2)^5 = 1/32$

If 5 cows bred, what is the probability of getting 3 bulls and 2 heifers?

a = bull; b = heifer; 5 reps ➔ $10a^3b^2 = 10(1/2)^3(1/2)^2 = 10(1/8)(1/4) = 10/32$

PRACTICALITY: **PROGENY TESTS**

The rules of probability can be used to calculate the number of matings necessary to determine whether breeding animals of NORMAL phenotype are **CARRIERS** of a **recessive gene.** Are these individuals homozygous or heterozygous?

TEST 1: Breed a "normal" bull to a dwarf cow.

Normal = DD or Dd; Dwarf = dd

# of matings	# independent events	Probability of normal calves	
1	$(1/2)^1$	½	0.50
2	$(1/2)^2$	¼	0.25
3	$(1/2)^3$	1/8	0.125
4	$(1/2)^4$	1/16	0.063
5	$(1/2)^5$	1/32	0.031
6	$(1/2)^6$	1/64	0.016
7	$(1/2)^7$	1/128	0.008
8	$(1/2)^8$	1/256	0.004
9	$(1/2)^9$	1/512	0.002
10	$(1/2)^{10}$	1/1024	0.001

If we breed a normal bull to 10 dwarf cows and produce 10 normal calves, the probability of the bull being a carrier (heterozygous) is less than 1 in a 1000. We are 99.9% confident that the bull is homozygous.

TEST 2: If we cannot identify a homozygous recessive individual, but CAN identify carrier individuals, we can mate an unknown bull to known carriers and look at offspring: D? X Dd

# of matings	# independent events	Probability of normal calves	
1	$(3/4)^1$	¾	0.75
2	$(3/4)^2$	9/16	0.56
3	$(3/4)^3$	27/64	0.42
x	x	x	x
10	$(3/4)^{10}$	59049/1048576	0.06

If the bull produces 10 normal offspring, then the possibility of the bull carrying the dwarf gene is 6/100. We are 94% confident that the bull is homozygous.

What if the bull produces 1 dwarf calf?

PRACTICALITY: **PREDICTION OF OFFSPRING**

The rules of probability can be used to PREDICT the phenotypes of offspring **IF** the GENOTYPES of the parents are known.

If we mate 2 heterozygous blacks, what is the probability of each of the 3 possible genotypes?

SIRE	DAM
Bb	Bb

Probability of receiving a B from sire = $\frac{1}{2}$
Probability of receiving a B from dam = $\frac{1}{2}$
*Probability of the offspring being BB = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

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Probability of receiving a b from sire = $\frac{1}{2}$
Probability of receiving a B from dam = $\frac{1}{2}$
*Probability of the offspring being bB = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Since Bb and bB are phenotypically alike, the probability of the Bb genotype is $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

Probability of receiving a b from sire = $\frac{1}{2}$
Probability of receiving a b from dam = $\frac{1}{2}$
*Probability of the offspring being bb = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Therefore, the genotypic ratio is 1BB: 2Bb: 1bb.

*If B is dominant, the phenotypic ratio will be 3 black: 1 red.
We would predict that 75% of all offspring will be black.

What if we breed a black bull to 4 black cows and get 4 red calves? Is this possible?
What are the odds?
$(\text{black} + \text{red})^4 = \text{black}^4 + 4\text{black}^3\,\text{red} + 6\text{black}^2\,\text{red}^2 + 4\text{black red}^3 + \text{red}^4$

$\text{red}^4 = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = 1/256 = 0.4\%$
$\text{black}^4 = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = 81/256 = 32\%$