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6.3 Set Theory: An Overview

Pitch Classes

Pitch class = A category of notes including all possible octaves and enharmonic spellings of a particular pitch. There are 12 pitch classes. For instance, C, B#, and D \flat in *any octave* are all in one **pitch class**.

Pitch classes are normally numbered from 0 to 11, starting on C:

Pitch class numbers: 0 1 2 3 4 5 6 7 8 9 "T" "E"
 =10 =11

Transposition of Sets (T_n)

Set = Any unordered group of unique (no-repeats) pitch classes. For example, D, F, A# is (2, 5, T).

To transpose a set **up** by *n* half steps, **add n** to each pitch class in the set.

To transpose a set **down** by *n* half steps, **subtract n** from each pitch class in the set.

If you get a number larger than 11 or smaller than 0, **add or subtract 12** to get a valid pitch class number.

Example showing transposition up 3 half steps:

Original: Pitch class: 4 9 1 2
 New pitch class: 7 0 4 5
 (Calculation:) (4+3=7) (9+3=12) (1+3=4) (2+3=5)
 12-12=0

Example showing transposition down 3 half steps:

Original: Pitch class: 4 9 1 2
 New pitch class: 1 6 T E
 (Calculation:) (4-3=1) (9-3=6) (1-3=-2) (2-3=-1)
 -2+12=10) -1+12=11)

Inversion of Sets (I)

Inversion (I) = reverse the *direction* of the intervals, keeping the number of *half steps* the same.

To invert a set, **subtract each pitch class number from 12** (i.e., 12-x).

If you get a number larger than 11 or smaller than 0, **add or subtract 12** to get a valid pitch class number.

Original: P4 up M3 up m2 up
 Pitch class: 4 9 1 2
 Inversion (I): P4 down M3 down m2 down
 New pitch class: 8 3 E T
 (Calculation:) (12-4=8) (12-9=3) (12-1=11) (12-2=10)

Transposing and Inverting (T_nI) Shortcut (OPTIONAL)

To do transposition by any number of half steps *n* AND invert the set, you can transpose, then invert, **OR** an *optional* shortcut for transposing and inverting *all at once* is **(n-x)** for each pitch class,

where *n* is the number of half steps to transpose and *x* is the original pitch class number.

If you get a number larger than 11 or smaller than 0, **add or subtract 12** to get a valid pitch class number.

Example showing T₂I:
 Pitch class: 4 9 1 2
 T₂I: (compare inversion ex. above)
 New pitch class: T 5 1 0
 (Calculation:) (2-4=-2) (2-9=-7) (2-1=1) (2-2=0)
 -2+12=10) -7+12=5)

6.3 Set Theory: An Overview, cont.

Normal Order

Normal order = (1) Pitches written low to high in 1 octave, **and** (2) compact as possible from bottom to top, **and** (3) the smallest intervals are placed as far toward the bottom as possible.

STEP ONE: Remove duplicated notes and reorder from low to high. You can start on any of the notes.

Given notes:

STEP TWO: Select the starting note that makes the set as compact as possible from bottom to top.

STEP THREE: *Sometimes* two options are tied after step two. If so, **break the tie** as follows:

1. Choose the option with the **smallest** interval from the **first** note to the **second-to-last** note.
2. If there is still a tie, choose the option with the **smallest** interval from **first** to the **third-to-last** notes.
3. If the tie cannot be broken after comparing all the intervals, the set is **transpositionally symmetrical**. In that case, choose the option with the **lowest starting pitch class number** (that is, closest above C).

Best Normal Order

The **best normal order** of a set is *either* the set's normal order **OR** the normal order of the set's *inversion*, whichever has the *smaller* intervals closer to the bottom.

To find best normal order:

1. Find the normal order of the *original set*.
2. Invert the set (see *Inversion of Sets*, previous page).
3. Find the normal order of the *inversion*.
4. Select *either* the original normal order **OR** the normal order of the inversion using "Step Three" of the normal order procedure above. If there is a tie at this point, the set is **inversionally symmetrical**.

Transpositional and Inversional Symmetry

A set is **transpositionally symmetrical** if its normal order divides the *octave* with a *repeating* pattern of half steps, like 1-5--1-5 or 1-3-2--1-3-2 or 4-4-4. The repeating pattern means you can start on different note(s) in the set and keep the pattern. These sets always have an unbreakable tie for **normal order**.

A set is **inversionally symmetrical** if it can be arranged to divide the *octave* with a pattern of half steps that is **palindromic** (the same forwards and backwards), like 1-5-5-1 or 3-6-3 or 4-4-4. This type of pattern means you can build the set upwards or downwards (=inversion) and get the same interval pattern.

Transposing the inversion by some interval will give you back the original notes.

These sets always have an unbreakable tie for **BEST normal order**.

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6.3 Set Theory: An Overview, cont.

Best Normal Order Example

1. Find the *normal order* of the set:

Given notes: \rightarrow Reorder low to high and remove duplicates: \rightarrow Select the most compact arrangement:

Starting on E: 4 half steps **NORMAL ORDER** | Starting on G: 9 half steps | Starting on $A\flat$: 11 half steps

2. Find the normal order of the *inversion*:

Normal order of original set from step 1: \rightarrow Find inversion using $(12-x)$: \rightarrow Reorder low to high, no duplicate notes: \rightarrow Select most compact arrangement:

Pitch classes: 4 7 8 \rightarrow 8 5 4 \rightarrow $=12-4 =12-7 =12-8$

Starting on E: 4 half steps **NORMAL ORDER** | Starting on F: 11 half steps | Starting on $A\flat$: 9 half steps

3. Select the "best" of the two normal orders by comparing intervals. Since the outside interval (bottom to top) is the same, choose the smallest interval from *first* note up to the *second-to-last* note:

Normal order of original set: 3 half steps, 4 half steps *versus* Normal order of inversion: 1 half step, 4 half steps

BEST NORMAL ORDER
since $E-F$ is 1 half step and $E-G$ is 3 half steps and $1 < 3$

Prime Form

Prime form = the best normal order as pitch class numbers transposed to start on C. Since $C=0$, prime form reveals the *number of half steps* each pitch class is *above* the first one. Prime form provides an easy shorthand for comparing the interval structure of sets throughout a piece of music.

Best normal order (from previous example): \rightarrow Transpose to C to find prime form: \rightarrow PRIME FORM = (014)

Pitch classes: 4 5 8 \rightarrow 0 1 4 $=4-4 =5-4 =8-4$

1 half step, 4 half steps

In this case, the prime form is (014); the interval structure of the best normal order is thus 1 half step above the first pitch and 4 half steps above the first pitch: (014).

Format for Writing Sets as Text

Pitch class set (unordered): Use parentheses and commas, e.g. (E,G, $A\flat$)

Normal order (or "normal form") OR best normal order: Use brackets & commas, e.g. [E,F, $A\flat$]

Prime form: Parentheses, NO commas, e.g. (014). Use "T" for ten and "E" for eleven.